A Numerical Analysis of the Dynamic Behaviour of a Composite Rotor Considering Its Sequential Damage Process

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ABSTRACT

Composite materials offer, in comparison to classical materials, high strength and stiffness to weight ratios, adjustable directional material properties and gradual damage behaviour. Due to such outstanding properties, a growing interest to use composite materials in weight-relevant applications of complex loaded structures is noticeable. Especially in the aerospace industry, application of composite materials is observed in the area of high-speed rotors.

One of the factors that affect the operational capability of rotors is unpredictable damage caused by impact events. This factor has a stochastic and discrete influence on the damage evolution of the rotors, which consequently affect their reliability and lifetime. Therefore, research efforts towards the development of reliable procedures for instantaneous identification of damage evolution are required. One of the most important tasks for a reliable structural integrity identification method is the selection of appropriate damage-dependent features, which could be measured in future generations of rotors with material-integrated sensors. In the presented work, a carbon fibre-reinforced epoxy disc rotor with its typical damage behaviour is numerically investigated.

In the case of composite materials, the consideration of failure as a sequence of distinct, physically based damage events depending on the damage behaviour of the composite material could provide additional knowledge in comparison to the common consideration of isolated damage events. To achieve this additional knowledge, a relation between damage evolution sequence and the damage-dependent dynamic response could be identified in order to diagnose with a higher accuracy the current damage condition compared to analyses of isolated damage conditions.

In order to acquire the damage-dependent vibration-based features, a numerical model based on the finite element method, coupled with a damage model implementing novel failure criteria based on the Failure-Mode-Concept, is created. Then, a numerical experiment corresponding to a practice relevant damage scenario was performed for a basis disc rotor made of carbon fibre-reinforced epoxy. The rotor with simulated impact-caused initial damage at different position for every test case was accelerated to the operational speed while the increasing body forces caused the initial damage to develop. The occurring damage evolution sequence was calculated using the implemented damage model. The damage-dependent dynamic response of the rotor consisting of its natural frequencies is analysed using the finite element method for each characteristic stage of the damage evolution process. A databank of multiple sequences of damage sensitive parameters is created that can be used for the similarity estimation between dynamic response sequences using distance factors in order to identify the damage condition of the rotor.

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INTRODUCTION

Composite materials offer, in comparison to classical materials, high strength and stiffness to weight ratios, adjustable directional material properties and gradual damage behaviour. Due to such outstanding properties, a growing interest to use composite materials in weight-relevant applications of complex loaded structures is noticeable. Especially in the aerospace industry, application of composite materials is observed in the area of high-speed rotors.

One of the factors that affect the operational capability of rotors is unpredictable damage caused by impact events. This factor has a stochastic and discrete influence on the damage evolution of the rotors, which consequently affect their reliability and lifetime. Therefore, research efforts towards the development of reliable procedures for structural integrity identification are required [1].

State of the art

One of the most important tasks often encountered in structural integrity identification methods is the selection of appropriate and sufficient damage-dependent features, e.g. modal properties [2]. In many cases, the complexity of the structure and the operational conditions result to a huge number of structural states which, in order to be accurately identified, dictate the use of a great amount of features. In most cases this is not possible due to limitations resulting from the measuring technology, the designer's knowledge of the system and other influencing parameters. Therefore, given a finite number of features, the goal is to develop algorithms that extract damage-relevant information and deliver the largest possible diagnostic accuracy.

A common approach in structural integrity identification methods is to conduct isolated vibration-based integrity analyses at different time points in the lifetime of the structure [3]. For example at Kostka et al. [4,5,6] the structural integrity of a carbon fibre-reinforced epoxy plate was investigated where damage was replicated by the attachment of a small mass or a damping layer in order to simulate the alterations of mass and damping characteristics and the resulting changes of the modal properties (e.g. eigenfrequencies). A problem encountered was the measurement of similar feature values for different positions of the attached mass or for different masses. This problem led to a misclassification of investigated examples.

Early investigations performed at a glass fibre-reinforced epoxy disc rotor revealed a similar problem [7]. In the case of rotors, the modal properties are not only dependent from the mass, stiffness and damping characteristics but also from the rotational velocity. It has been observed that for a range of rotational speeds the eigenfrequencies of the rotor containing different damages were of similar values and close to the variance from the measuring systems [6]. The use of those test cases could have reduced the percentage of accuracy of the diagnostic classification methods. The complex damage-dependent alterations of the anisotropic stiffness, leading also to coupling effects, can result in the appearance of similar feature values for different damage conditions.

In the case of composite materials, failure is typically not considered as a unique event but as a gradual sequence of matrix cracking, delamination, fibre-matrix debonding, shear nonlinearities and fibre failure leading up to structural collapse. The operational capability of a structure can be maintained whilst damage evolves in the structure up to a specific damage threshold. Nevertheless, the consideration of the past damage events and the gradual damage behaviour of these materials in structural integrity identification methods is not yet considered.

For the calculation of the damage evolution under complex loading conditions reliable damage mechanics models based novel failure criteria is developed [8]. These models use the Failure Mode Concept (FMC) which take into account the material-symmetries by the application of invariants. A unidirectional lamina is homogenized to a 'material' and a separate strength criterion is allocated to one failure mode and to one associated basic strength. It focuses on two aspects of the theoretical prediction of failure in composites; the first is the derivation of failure conditions for a unidirectional lamina with the prediction of initial failure of the embedded lamina, and the second is the treatment of non-linear, progressive failure of 3-dimensionally stressed laminates until final failure.

Based on the FMC, the following failure modes can be identified:

- fibre final tensile/compressive failure,
- delamination,
- matrix-tensile/compressive/shear failure,
- fibre-matrix shear failure,
- material damage (shear nonlinearity).

Physically-based damage events described using the FMC allow the modelling of damage evolution of composite materials as a continuous process that can be discretised and considered as a sequence of distinct events. Existing methods of machine learning, [9], could be used for the analysis of damage evolution sequences of composite materials.

Problem definition and proposed approach

The problem of similar feature values for different damage cases can be described as the lack of the bijectivity property of the function $y = f(x)$ that maps the relationship between the state of damage and a diagnostic feature. Such feature can describe the dynamic behaviour of the structure, e.g. modal properties, which is shown qualitatively in Fig. 1. As a consequence, a diagnostic feature value y_o corresponds to multiple discrete states, x, and therefore even if the function $y = f(x)$ can be analytically described, it cannot be inverted.

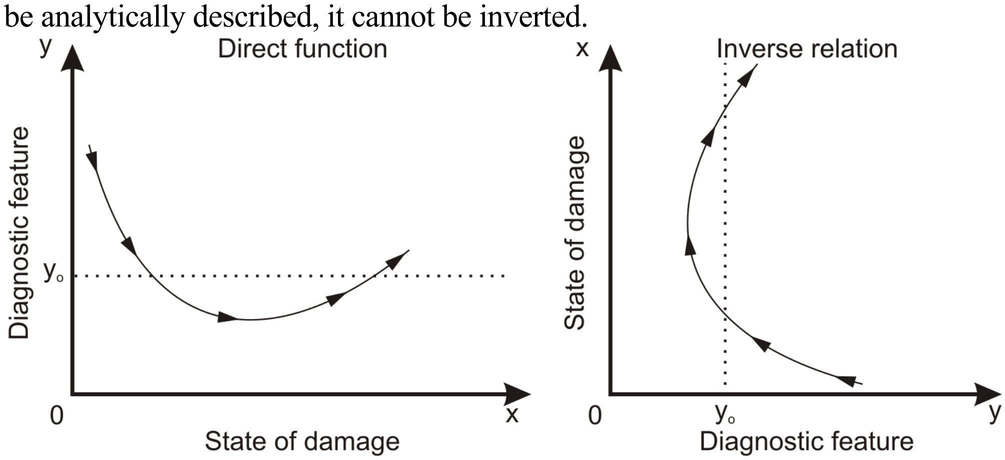

Figure 1. Schematic view of the lack of the bijectivity property for the function describing the relationship between the state of damage and a diagnostic feature.

To avoid this problem, the proposed approach considers past events in the form of a sequence of features that are sensitive to physically occurring damage phenomena being of sequential nature in the case of multilayer composites. Instead of typical analysis of isolated sets of feature values corresponding to the examined damage condition, the similarity between the available sequence feature values collected on the examined object and known exemplary sequences is evaluated. The presented work was based on numerical simulations of the damage-dependent dynamic behaviour of a composite rotor. In such application, a sequential damage evolution behaviour due to an initial damage and subsequent load resulting from body forces is typical. The here proposed SHM-approach for composite rotors consists of the following steps:

1. Sequences of damage evolution representative for the analysed application are collected, e.g. using appropriate simulation models such as FMC. Simultaneously, the damage-dependent material properties and the measurable features of the structural dynamic behaviour are calculated.
2. During the operational lifetime of the structure:
   - values of features are collected describing the structural dynamic behaviour,
   - methods are applied for the similarity estimation between the already collected sequence of damage-sensitive features and the previously calculated sequences (point 1),
   - an association is performed between the collected sequence and one of the representative sequences in order to estimate both the current damage condition and its probable evolution.

The here presented work considers mainly the development of an approach for the similarity estimation between sequences of damage-sensitive features.

DAMAGE EVOLUTION AND DYNAMIC RESPONSE SEQUENCE

Consider an example of a **damage evolution sequence** of a composite structure as an arbitrary sequence X describing m considered discrete damage states, $X = \langle \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m \rangle$. Each term $\mathbf{x}_i$ of the sequence is a matrix, Equation (1), corresponding to a damage state of the structure. Every term FM_i describes the existence of a failure mode (FM) at k separated volume elements of the structure. In the case of previously described FMC, five failure modes are considered, thus $\mathbf{x}_i$ has a size of $k \times 5$.

$$\mathbf{x}_i = \begin{bmatrix} FM_i^{1,1} & FM_i^{1,2} & \cdots & FM_i^{1,5} \\ FM_i^{2,1} & FM_i^{2,2} & \cdots & FM_i^{2,5} \\ \vdots & \vdots & \ddots & \vdots \\ FM_i^{k,1} & FM_i^{k,2} & \cdots & FM_i^{k,5} \end{bmatrix} \quad (1)$$

Consider the damage-dependent dynamic behaviour during the operational time as a **dynamic response sequence** Y of the structure, $Y = \langle y_1, y_2, \ldots, y_m \rangle$, where each term y_i of the sequence is a vector which includes the p eigenfrequencies (EF), $y_i = \{EF_i^1 \quad EF_i^2 \quad \cdots \quad EF_i^p\}^T$ corresponding to the damage state $\mathbf{x}_i$ from the sequence X. Let the ordered pair (X_j, Y_j) be an example of the damage evolution sequence of a structure and its corresponding dynamic response sequence, respectively, and $\mathbf{D} = \begin{Bmatrix} X_1, Y_1 \\ X_2, Y_2 \\ \vdots \\ X_n, Y_n \end{Bmatrix}$, a set of n different examples of sequences created from numerical- or experimental-based test cases.

Consider an unknown dynamic response sequence Y_u, where $Y_u \notin \mathbf{D}$, $Y_u = \langle y_{u,1}, y_{u,2}, \ldots, y_{u,k} \rangle$ and $1 \leq k \leq m$. The goal is to find the most similar sequence $Y_j = \langle y_{j,1}, y_{j,2}, \ldots, y_{j,k} \rangle \in \mathbf{D}$ and then, given (X_j, Y_j), to associate Y_u with the corresponding damage sequence $X_j = \langle \mathbf{x}_{j,1}, \mathbf{x}_{j,2}, \ldots, \mathbf{x}_{j,k} \rangle \in \mathbf{D}$. As a result, not only the probable damage condition of the structure can be identified in this way, but also a prognoses of the future damage evolution $X_j = \langle \mathbf{x}_{j,k+1}, \mathbf{x}_{j,k+2}, \ldots, \mathbf{x}_{j,m} \rangle$.

DISTANCE FACTORS FOR THE SEQUENCE SIMILARITY

The proposed method aims to find the degree of similarity between sequences of damage-sensitive features using multiple distance factors, where each factor is sensitive to different characteristics of the sequences [10]. Let Y_u be an unknown dynamic response sequence and Y_s a known sequence from the set $\mathbf{D}$, with $1 \leq s \leq n$.

The first distance factor is based on the Standardized Euclidean Distance $S_{dist,1}$ between the sequences where

$$S_{dist,1} = \sqrt{(Y_s - Y_u)V^{-1}(Y_s - Y_u)^T} \tag{2}$$

V is the $k \times k$ diagonal matrix whose z^{th} diagonal element is $S(z)^2$, and S the vector of the standard deviations [11].

The second distance factor is based on the sample correlation with

$$S_{dist,2} = 1 - \frac{(Y_s - \bar{y}_s)(Y_u - \bar{y}_u)^T}{\sqrt{(Y_s - \bar{y}_s)(Y_s - \bar{y}_s)^T}\sqrt{(Y_u - \bar{y}_u)(Y_u - \bar{y}_u)^T}} \tag{3}$$

where $\bar{y}_s = \frac{1}{k}\sum_j y_{sj}$ and $\bar{y}_u = \frac{1}{k}\sum_j y_{uj}$ [11].

The joint distance factor DF is calculated through the summation of the $S_{dist,1}$, $S_{dist,2}$ and is a vector that expresses distance measure between each two corresponding terms of the investigated sequences.

$$DF = a_1 S_{dist,1} + a_2 S_{dist,2} \tag{4}$$

and a_1, a_2 parameters that provide different weights to the two distance factors based on the desired sensitivity for each of the factors.

The DF_T is the accumulated distance factor based on the DF,

$$DF_T = \sum_{i=1}^{k} DF_i \,. \tag{5}$$

The DF_T could be directly used during the SHM process for the assessment of the similarity between the measured dynamic response sequence and known reference sequences.

NUMERICAL MODEL AND SIMULATION PROCEDURE

A numerical experiment corresponding to a practice-relevant damage scenario is performed for a basis disc rotor made of carbon fibre-reinforced epoxy, Fig. 2(a). In the presented work, only combinations of failure types observed after impact-caused damages are considered [12], in particular delamination, matrix failure and fibre-matrix shear failure.

A finite element (FE) model of the rotor is created and selected degrees-of-freedom are constrained in order to simulate fixed boundary conditions at the inner circular cut-out of the investigated rotor. The position and size of the simulated impact-caused initial damage were varied in order to create different test cases depending only on the position and size of the damage. The rotor with and without the simulated initial damage was accelerated to a defined operational speed of 20.000 rpm while the increasing body forces caused the initial damage to develop, and the occurring damage evolution sequence was calculated using the damage mechanics model based on the FMC. The damage-dependent dynamic response of the rotor was calculated for each characteristic stage of the damage evolution process.

Input data for the FE model

The investigated basis disc rotor has dimensions shown in figure 2(a) and its composite layup is shown in figure 2(b). The material properties of the carbon fibre-reinforced epoxy and the required parameters for the FMC criteria [8] are extracted from the material database of the Institute of Lightweight Engineering and Polymer Technology, TU Dresden.

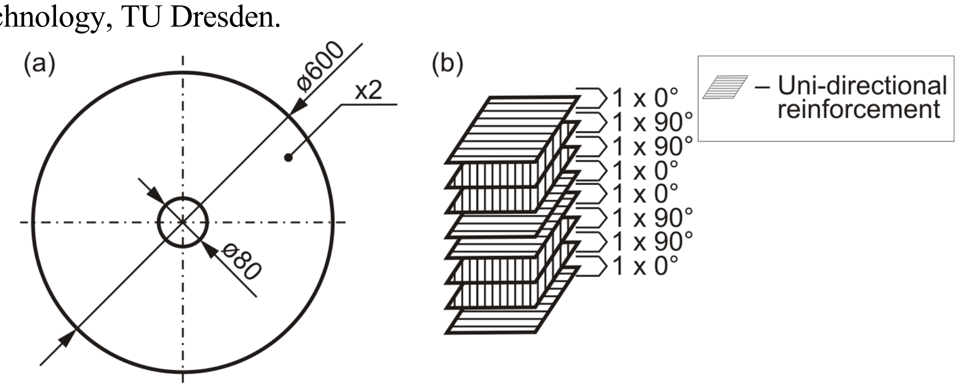

Figure 2. Dimensions of the rotor (a), and composite layup of the rotor (b).

Simulation procedure for the generation of sequences

The rotor run up with an angular acceleration of 50 $rad/\sec^2$ in a range from 0 rpm to 20.000 rpm was simulated. At each 500 rpm, the failure modes at each element were estimated using the damage mechanics model in order to form the damage evolution sequence X_i that describes the damage evolution process of the rotor. In addition, the modal properties, i.e. first 10 eigenfrequencies, were calculated using the FE model in order to form the dynamic response sequence Y_i.

RESULTS

The percent of the accumulative matrix failure for four different test cases is shown in figure 3(a), which was calculated from the respective damage evolution sequences X. Three test cases have an initial damaged area of 5%, and the position of the damage is varied (R: 100÷140 mm, $\theta: 0^\circ$) Fig. 3(b), while the fourth test case is without initial damage.

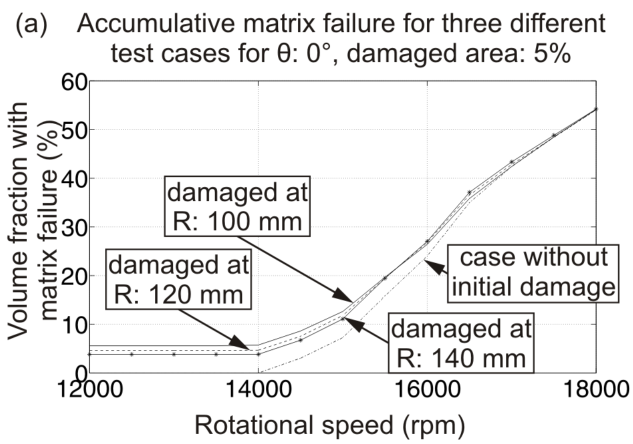

Figure 3. The accumulative matrix failure for four test cases calculated from the respective damage evolution sequences X (a), and the respective parameters for the position of the damaged area (b).

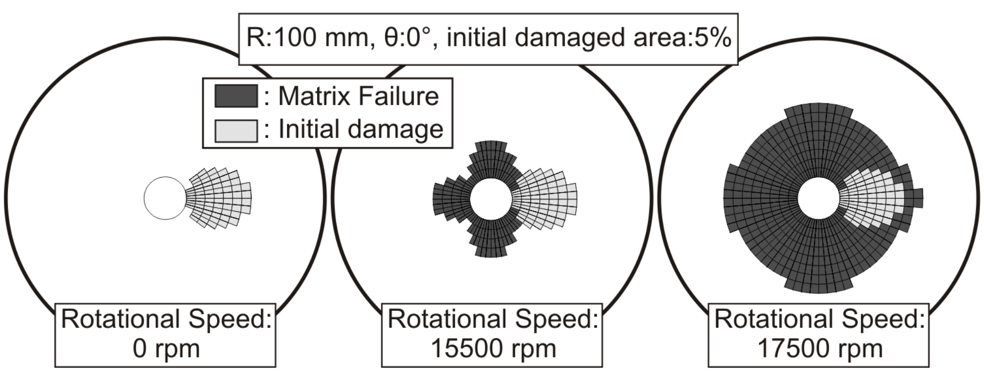

Figure 4. The damage evolution sequence of matrix failure for a test case at three different rotational speeds.

An example of the matrix failure evolution of the rotor is shown in figure 4 at three different rotational speeds for the test case (R: 100 mm, θ: 0°, initial damaged area: 5%). The initial damage is a combination of delamination, matrix failure and fibre-matrix shear failure. The respective parameters for the position of the damaged area are shown in figure 3(b).

An example of the distance factor DF is shown in figure 5 for three different test cases. The test case without initial damage is considered as the reference case. The distance factors were calculated between dynamic response sequences of each test case and the reference case. The presented results were obtained with the parameters a_1, a_2 equal to unity. The accumulated distance factor DF_T for the three test cases (R: 100 mm, 120 mm and 140 mm, θ: 0°, initial damaged area: 5%) is 4.74, 2.52 and 1.94, respectively. Based on the DF_T, the third test case shows the highest similarity to the reference case without initial damage, R: 140 mm, where the damaged area is located the furthest from the centre of the rotor. This is to be expected as damage located at the outer area of the rotor has a smaller influence to the damage evolution sequence and the resulting dynamic behaviour in comparison with damage inflicted close to the clamping area.

The evolution of damage begins roughly at 14.000 rpm as shown in figure 3(a). However, a strong increase of the distance factor can be observed earlier, i.e. at 11.000 rpm for all three test cases, Fig. 5. The increase of the distance factor results from the existence of the initial damage, and therefore, it can be used as information for the identification of the damage state of the rotor.

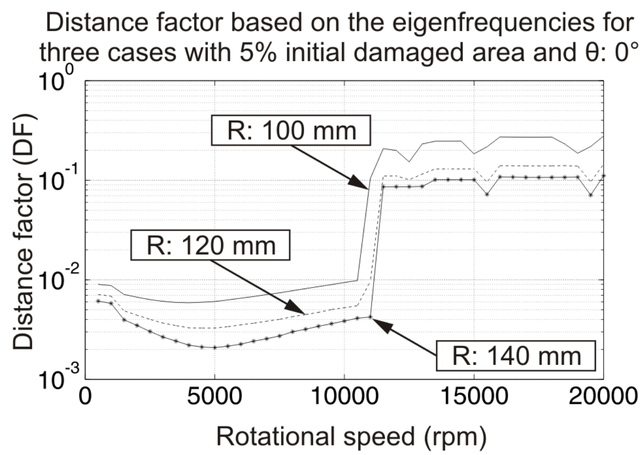

Figure 5. Distance factor based on the eigenfrequencies between three test cases and a reference test case of the rotor.

CONCLUSION AND DISCUSSION

A basis disc rotor made of carbon fibre-reinforced epoxy was numerically investigated regarding a practice-relevant damage scenario with simulated impact-caused initial damage. A damage mechanics model was combined with a FE model in order to generate sequences of damage evolution and the corresponding dynamic response sequences of the rotor. The here presented work considered an approach for the similarity estimation between dynamic response sequences through the implementation of distance factors. The obtained results show a high sensitivity of these factors to the evolving damage.

The reliability of the proposed approach must be additionally examined with different damage cases. The influence of the a_1, a_2 parameters on the similarity estimation should be also investigated. The robustness of the approach should be considered through the introduction of noise to the calculated features, based on typical signal-to-noise ratios of the existing measuring devices.

The here applied sampling of damage evolution and dynamic response sequences that is controlled by the rotational speed during the rotor run up was used only for the numerical investigation of the similarity between different damage evolutions resulting from the composite properties, initial damage and variable load conditions. The sampling of sequences should be in practical applications controlled by relevant changes of the damage condition. This problem will be addressed in future investigations.

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