Lamb Wave Propagation Modeling Using Cellular Automata

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ABSTRACT

Theory of Lamb Wave propagation in plate-like structures have found many practical applications in Structural Health Monitoring. However for better understanding of complex physical phenomena associated with wave propagation and wave interaction with damage numerical simulations are as important as laboratory experiments. The paper shows the application of Cellular Automata technique for modeling of elastic wave propagation. After a brief introduction to Lamb waves, 2-D triangular Cellular Automata approach for wave propagation is presented. Numerical simulations are performed for undamaged and damaged aluminium plates. The results are compared with the Local Interaction Simulation Approach (LISA).

INTRODUCTION

Monitoring for structural damage is important in maintenance of many engineering structures. Various damage detection techniques have been developed for Structural Health Monitoring (SHM) applications. Methods based on guided waves are particularly attractive in plate-like structures. It appears that Lamb waves are the most widely used guided ultrasonic waves for damage detection in metals and composites, as reviewed in [1-3]. Despite numerous laboratory implementation SHM application of Lamb waves in real engineering systems remain limited. The complexity of physical mechanism associated with these waves is an important factor when implementation of Lamb waves techniques is considered for damage detection.

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Also, signal features produced by defects tend to be embedded in the background noise associated with material, structural and environmental variability and thus are difficult to detect reliably. It is widely acknowledged that numerical simulations can help with the entire damage detection implementation process and interpretation analysis, leading to accurate SHM diagnosis and prognosis.

Various methods have been developed for modelling and numerical simulations of elastic wave propagation, including semi-analytic techniques (e.g. methods based on the theory of diffraction, boundary element methods) and numerical algorithms such as methods based finite differences, finite elements, spectral elements, elastodynamic finite integration techniques (EFIT) or local interaction simulation approach (LISA), as reviewed in [4] . The LISA is particularly attractive for very fast parallel computation of large models of wave propagation in complex media with sharp interfaces [5]. More recently the Cellular Automata (CA) approach based on rectangular [6] and triangular [7] . The latter offers arbitrary meshing geometries and is an attractive alternative finite difference based methods.

The major objective of the paper is to explore the CA approach for Lamb wave propagation in metallic structures. A 2-D case study of wave propagation in undamaged and damaged aluminium plates is investigated. Numerical results are compared with simulations based on the LISA technique.

LAMB WAVES

Lamb waves are elastic perturbations propagating in solid plates with free boundaries. These waves arise from coupling of shear and longitudinal waves reflected at the top and bottom surfaces of the plate, leading to an infinite number of dispersive modes. The wave propagation problem can be analysed using the classical elastodynamic wave equation

$$(\lambda + \mu)\nabla\nabla \cdot W + \mu\nabla^2 W = \rho W_{,tt} \tag{1}$$

where where λ and μ are Lamé constants, ρ is the material density and W is the vector of particle displacements. This equation can be solved using the displacement potential approach or the partial wave technique, as demonstrated in [8]. The former decoupled the elastodynamic wave equations leading to the well known Rayleigh-Lamb frequency relations

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2 pq}{\left(q^2 - k^2\right)^2} \tag{2}$$

for symmetric S_n (n=0,1,2,...) modes

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{\left(q^2 - k^2\right)^2}{4k^2 pq} \tag{3}$$

for antisymmetric A_n (n=0,1,2, ...) modes, where k is the wave number and variables p and q can be defined in terms of circular frequency $\omega = 2\pi f$ and the shear c_T and longitudinal c_L wave velocities as

$$p^2 = (\omega/c_L)^2 - k^2 \qquad\qquad q^2 = (\omega/c_T)^2 - k^2 \qquad (4)$$

The fundamental S_0 and A_0 modes will only propagate in the plate for small values of the frequency-and-plate-thickness" product. The elastodynamic and Rayleigh-Lamb equations can be solved numerically to obtain physical displacements and phase/group velocities as a function of the frequency-and-plate-thickness" product , respectively.

CELLULAR AUTOMATA

Cellular Automata (CA) were introduced in the early 1940's and developed further in the following years by John von Neumann to model self-replicating systems [9]. At the same time Stanislaw Ulam started his work for Los Alamos Scientific Laboratory and began research related to biology [10]. Both scientists shared their experiences and built the base of the CA theory. Following these development the potential of the method for modelling complex physical phenomena has been recognised. Stephen Wolfram made a detailed study of CA and proposed their classification [11].

CA are mathematical idealisations of physical systems and introduce discretisation of time and space. CA consist of cells array in *D*-dimension. Every cell has set of states. These states are usually the same for all cells. The state of *i*-cell at a time step *t*+1 is determined by a rules function *R*. Set of rules utilize *i*-cell neighbours' state and its own state, i.e.

$$s_i(t+1) = R(\{s_j(t)\}),\ j \in N(i) \qquad (5)$$

where *R* is a function containing set of rules, s_i is the *i* cell's state in the next step, *t* is the time step, s_ji the neighbours' state in the previous step and *N*(*i*) is the neighbourhood of the *i* cell.

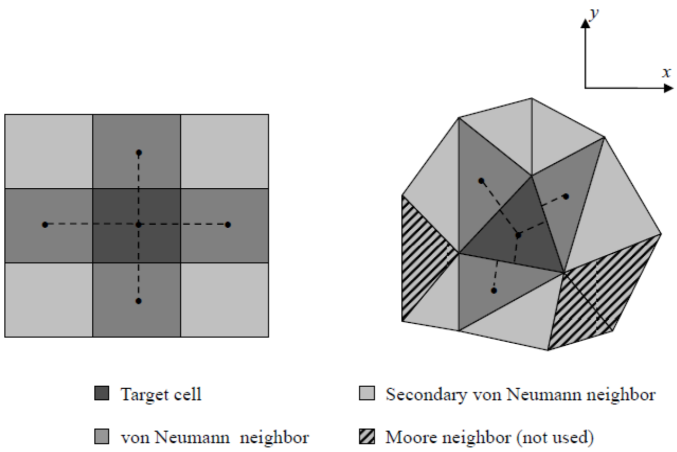

Figure 1. Neighbourhood structures in two-dimensional CA with comparison between classical rectangular automata (left) and triangular automata for arbitrary geometries (right) [12].

Two different types of cells are used in practice. Figure 1 shows neighbourhood types examples for rectangular and triangular CA. In these examples the dark cell in the middle of such structures is the *i*-cell from Equation (5) and can be described using the Cartesian coordinates pair (x, y) according to considered 2-D space.

CA are an efficient tool for dynamic physical systems simulations because of possibility of parallel computation. Numerical implementation of CA for mechanical phenomena simulation are different in comparison to methods based on finite elements that involve partial differential equation. In CA applications the global behaviour of physical system is determined by a local interaction between cells based on a specified rule set. Figure 2 shows how strains from neighbours acting on a particular cell can be calculated for the 2-D problem. These strains are used for specific rule functions introduced in Equation (5).

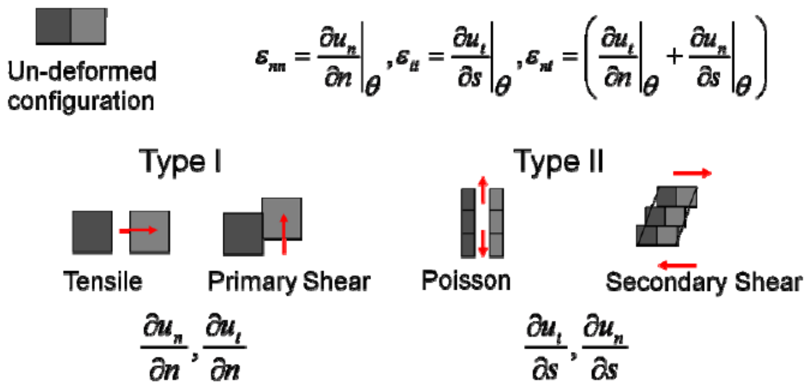

Figure 2. Strains needed to establish a rule set for CA [12].

LAMB WAVE PROPAGATION MODEL

CA were used to model Lamb wave propagation. A rectangular (400 x 150 mm; 2 mm thickness) aluminium plate (Young's modulus E = 71 GPa, Poisson ratio ν = 0.338 and density ρ = 2711 kg/m^3) was used in this model. Both damaged and undamaged cases were investigated. A circular hole (diameter equal to 4 mm), located in the middle of the plate, was considered as a simulated damage. Figure 3 illustrates the geometry of the considered model. The excitation and response positions were selected at the top (x=200 mm and y = 5 mm) and bottom (x = 200 mm and y = 145 mm) of the plate respectively.

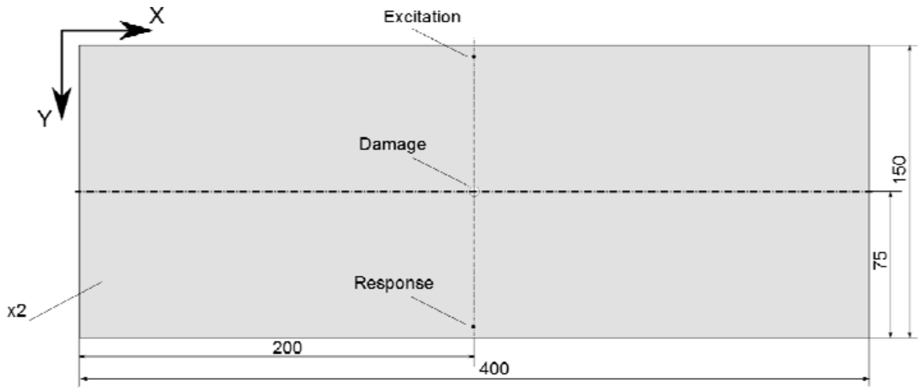

Figure 3. Aluminium specimen used for CA Lamb wave propagation modelling.

Parallel implementation of triangular CA - proposed and described in [12] - was used in these investigations. This implementation is based an object oriented code written in Java. The Java-based numerical simulation platform used allows for elastic wave propagation simulations. The geometry of the plate was modelled using a set of triangular cells obtained as a finite element mesh from the commercial COMSOL multiphysics software package. Figure 4 illustrates the mesh fragment for the plate with a circular defect. The meshes for the undamaged and damaged plates contained approximately 30 000 cells. The discretisation used was equal to 1e7 steps per second.

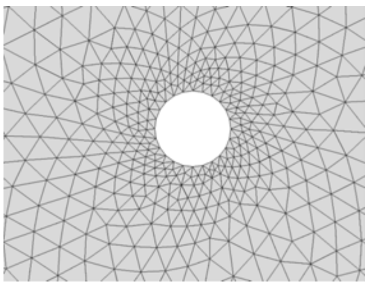

Figure 4. Mesh fragment for the plate with a circular defect.

A five-cycle - shifted by 90^0 sine burst signal with a Hann window envelope, as shown in Figure 5 - was used for excitation. Although in theory a minimum two Lamb wave modes propagate in plates, it is always possible to select excitation frequency for which the amplitude of one fundamental modes is reduced almost to zero. This so-called single mode excitation was used in the current 2-D investigations. The excitation frequency - selected experimentally - was equal to 100 kHz. This led to A0 Lamb wave mode propagation.

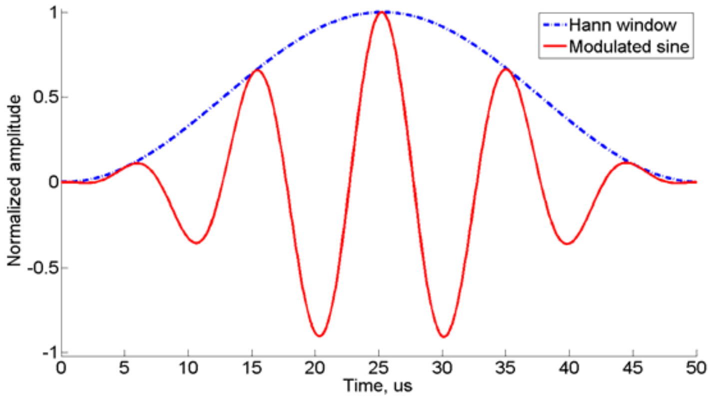

Figure 5. Excitation signal used for Lamb wave propagation modelling.

RESULTS AND DISCUSSION

Once the model of the undamaged and damaged aluminium plate implemented CA were used to simulated Lamb wave propagation. The simulation results for the (x,y) plane are presented in Figures was developed Figure 6.

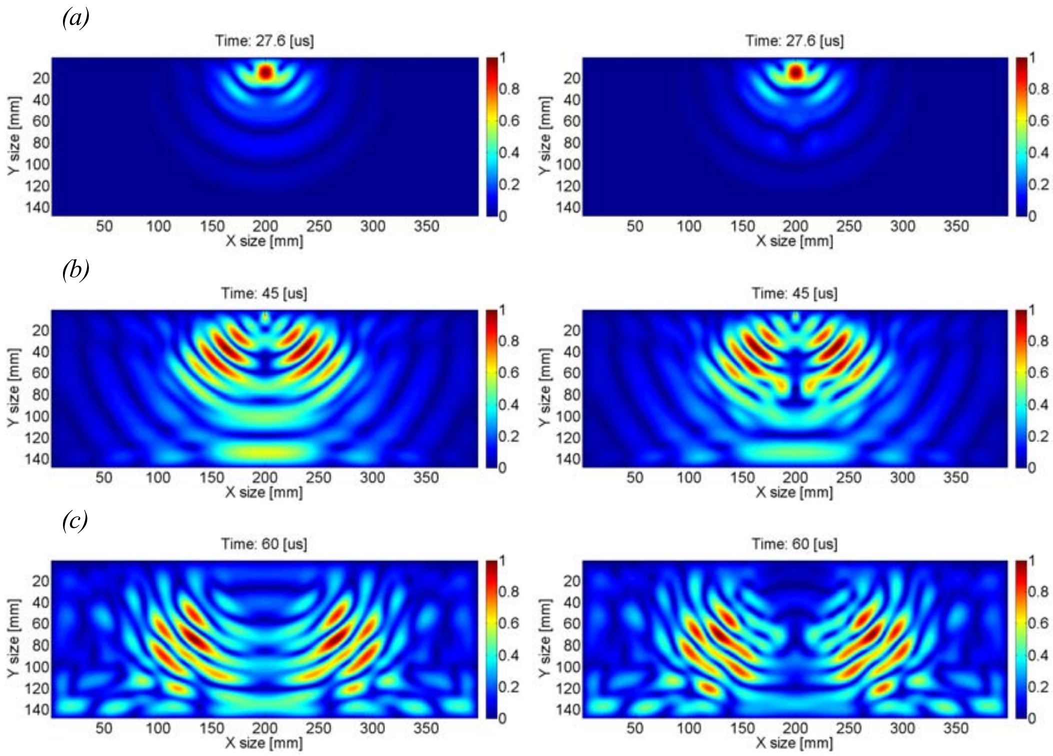

Figure 6. CA Lamb wave propagation simulated for the undamaged (left column) and damaged (right column) plate for various time snapshots.

Here, the normalised displacement amplitude is given for various time moments. The results - given for the undamaged plate in the left column and for the damaged plate in the right column - display expected wave propagation features. The incident wave is clearly broken after it passes through the hole (Figure 6a - right column and attenuated when it reaches the bottom of the plate (Figure 6b - right column. Numerous reflections from plate edges can be observed in Figure 6c for both cases investigated.

The same results but obtained for the LISA method [123] - shown in Figure 7 - display very similar wave propagation phenomena and wave interaction with damage features. Direct comparison of Lamb wave responses can be analysed in Figure 8. The CA and LISA results for the incident wave are almost identical for both cases investigated. However, wave reflection components simulated by the CA approach appear to be delayed if compared with the LISA approach. This is probably due to different meshing geometry used in the relevant models. The amplitude of the incident wave for the damaged plate is reduced in Figure 7b, as expected.

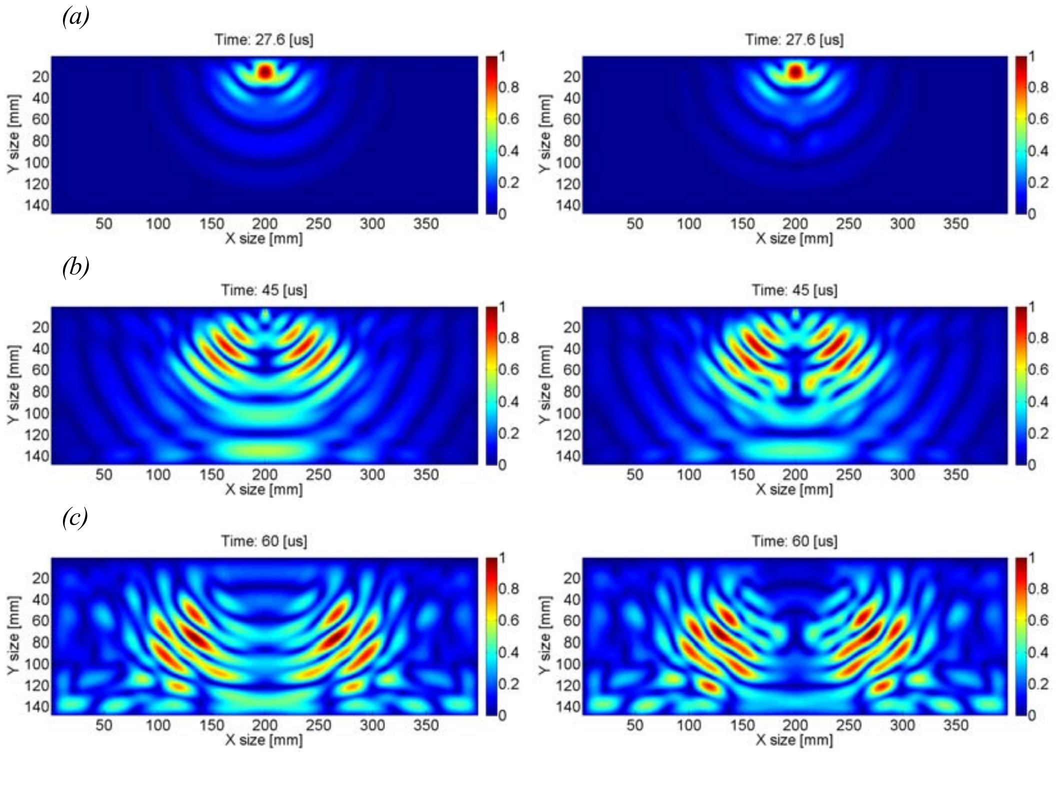

Figure 7. LISA Lamb wave propagation simulated for the undamaged (left column) and damaged (right column) plate for various time snapshots.

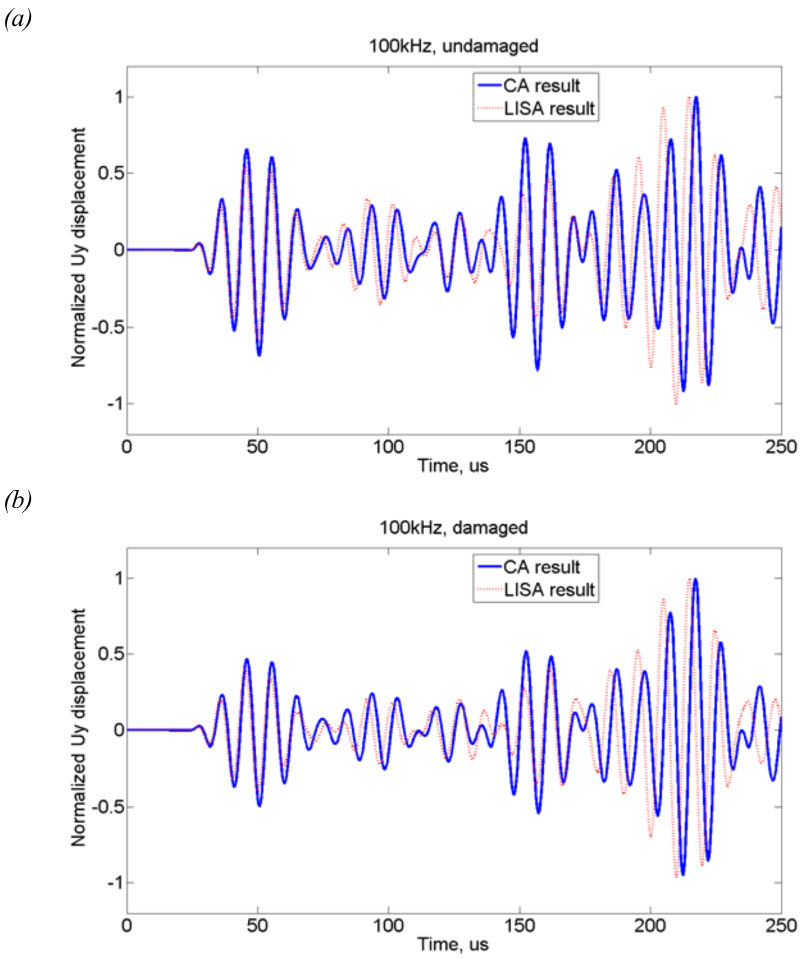

Figure 8. Lamb wave responses obtained for CA and LISA simulations: (a) undamaged plate; (b) damaged plate.

CONCLUSIONS

Cellular Automata were explored to simulate Lamb wave propagation in undamaged and damaged aluminium plates. A simple single-mode propagation was considered. The results were compared with the LISA method.

Wave propagation phenomena and damage features in Lamb wave responses were identical for both methods when compared visually. The amplitude and arrival time for the incident wave component were almost identical. However, some increased arrival times for reflected components could be observed for CA simulations if compared with the LISA results. These discrepancies - probably due to different meshing - need further investigations.

Nevertheless, these preliminary results are very promising. The ability of triangular CA to model Lamb wave propagation in arbitrary geometries has been demonstrated.

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