Damage Detection and Precise Localization via a Vibration Based Functional Model Method—Application to a 3D Truss Structure

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ABSTRACT

The goal of this study is the development of a statistical time series method for both damage detection and precise localization. The method is based on the postulation of Vector Functionally Pooled AutoRegressive with eXogenous excitation (VFP-ARX) models, where the operating parameter vector consists of three components, each one corresponding to a single dimension of the three dimensional space. The method essentially constitutes an extension of a simpler version working on a single dimension and recently introduced by the last two authors and their collaborators. The effectiveness of the method is experimentally assessed via several damage cases in a 3D truss structure and single-excitation single-response vibration signals.

INTRODUCTION

Vibration based methods have been used in various studies for damage detection and localization. These methods mostly belong to either the Finite Element [1] or the Statistical Time Series families [2]. The former family requires complete Finite Element models and the measurement of a sufficient number of vibration response signals. On the other hand, Statistical Time Series methods use identified models of small size – such as AutoRegressive Moving Average (ARMA) or State Space (SS) models – and a very limited number of measured signals (a single measurement may be sometimes sufficient). Yet, this family typically faces difficulties with the problem of damage localization. However there are methods, as for instance in [3], which offer the effective solution of damage classification in a range of pre-specified types of damages with certain locations and magnitudes on areas of the structure where arrays of sensors are mounted.

The *goal* of this study is the development of a Statistical Time Series method capable of achieving damage detection and precise localization on the continuous topology of a structure overcoming the limitation of damage classification mentioned above. The method is based on the postulation of recently introduced Vector

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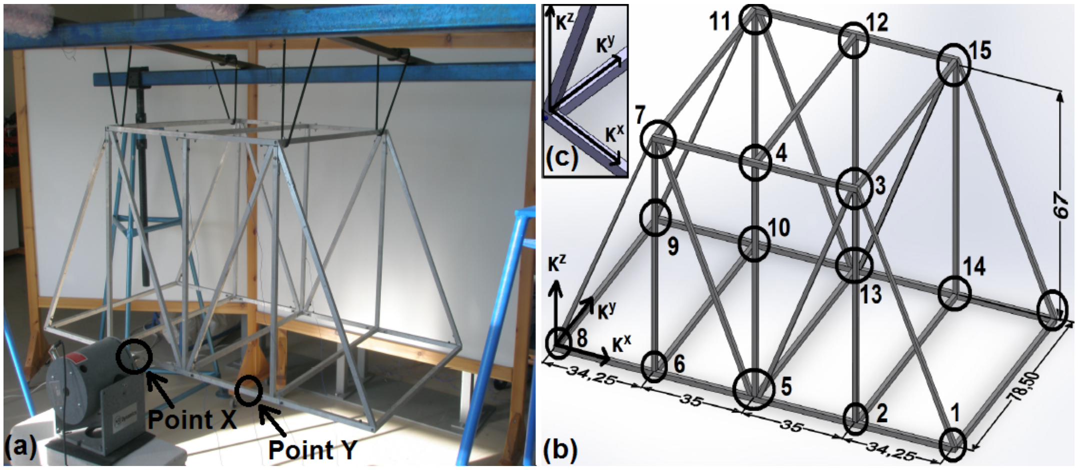

Figure 1. (a) The truss structure and the experimental setup: The force excitation (Point X) and the vibration acceleration measurement position (Point Y), (b) Schematic diagram of the truss. The 16 nodes are indicated by circles (dimensions are in cm) - the coordinate system origin is set at node 8, (c).

Functionally Pooled AutoRegressive with eXogenous excitation (VFP-ARX) models, where damage is represented on a continuous topology using a three dimensional Cartesian coordinate system. The method essentially constitutes an extension of a simpler version working on a single dimension which is presented in [4].

The effectiveness of the method is demonstrated through its use on damage detection and precise localization on a three dimensional (3D) aluminum truss laboratory structure which consists of 26 rods connected at 16 nodes via 74 bolts. Several damages are considered in a number of experiments, each one corresponding to the loosening of a different bolt at a time. Damage detection is based on a scheme of two non-parametric and one parametric technique and simple hypothesis testing procedures [2], while localization is based on estimation of a suitably re-parametrized VFP-ARX model.

THE EXPERIMENTAL SET-UP

The structure. The truss structure was designed and manufactured by the SMSA Laboratory in the University of Patras, Greece. It is suspended from metallic beams with a set of elastic cords and hooks [Figure 1(a)]. It consists of 26 rods with rectangular cross sections (1.5 $\times$ 1.5 cm) jointed together via steel elbow plates and bolts. The total number of bolts is 74. All parts are constructed from standard aluminum with the overall dimensions being 140 $\times$ 80 $\times$ 70 cm.

The damages and the experiments. Each considered damage corresponds to the complete loosening of a bolt with the total number of investigated damage cases being 74 (one for each bolt). The 74 bolts are distributed at 16 nodes as shown in Figure 1(b) – each node consists of closely located joint bolts – and each node has certain coordinates defined based on the origin point ($x = 0, y = 0, z = 0$) taken at node 8 [see Figures 1(b),(c)]. Thus, a damage is designated as F_{k^x,k^y,k^z}, with k^x, k^y, k^z the coordinates along the x, y, z axis, respectively.

Damage detection and localization are based on response only or single-excitation

TABLE I. EXPERIMENTAL DETAILS.

Structural State	Description	No of experiments
Healthy	-	10
Damaged	loosening of a bolt	Set A: 16 (one per node), Set B: 144 (9 $\times$ 16 bolt locations), Set C: 30 (10 $\times$ 3 bolt locations)
Sampling frequency: $f_s = 128$ Hz Signal Bandwidth: $[3 - 59]$ Hz (bandpass Chebyshev Type II; order 12)		
Signal length N in samples (s): Damage detection: $N = 9758$ (76.23 s) Damage localization: $N = 2500$ (19.53 s)		

single-response vibration signals. The y-directional excitation is a random Gaussian force applied horizontally at Point X [Figure 1(a)] via an electromechanical shaker equipped with a stinger and measured via an impedance head (PCB 288D01). The y-directional vibration acceleration is measured via a lightweight accelerometer (PCB ICP 352A10, 0.7 gr) at point Y [Figure 1(a)]. All signals are driven through a signal conditioning device into the data acquisition system (SigLab 20-42).

A number of experiments are carried out, initially for the healthy and subsequently for the damaged states of the structure. Experimental details are presented in Table I.

DAMAGE DETECTION AND LOCALIZATION

Damage detection and localization in a three dimensional structure may be based on the Functional Model Based Method (FMBM) [2, 4] by employing VFP-ARX models with four dimensional operating parameter vector. One dimension is for representing all admissible damage magnitudes and the other three correspond to the continuous topology representation of the 3D Cartesian system axis where a damage may occur. Yet, VFP-ARX models with three dimensional operating parameter vector is for the first time employed in this study as the damage in the truss structure has no different magnitudes (complete loosening of a bolt). By omitting the dimension of the damage magnitude, the FMBM is limited to damage localization on the 3D Cartesian system and for this reason damage detection is accomplished based on a scheme which employs the non-parametric Power Spectral Density (PSD) and Frequency Response Function (FRF) [2] as well as the parametric Sequential Probability Ratio Test (SPRT) based techniques [5].

Damage detection. In an initial *baseline phase*, the Power Spectral Density and the Frequency Response Function magnitude, which constitute the techniques' characteristic quantities, are estimated based on a single acceleration signal and a pair of excitation-response signals, respectively, corresponding to the healthy structure (see details in [2]). Similarly, the standard deviation of a model (herein a typical ARX) residual sequence corresponding to the healthy structure is the estimated characteristic quantity in the framework of the SPRT based technique.

In a subsequent *inspection phase*, damage detection is based on proper comparison of the current characteristic quantity of each technique to the characteristic quantity corresponding to the healthy structure. This is accomplished via binary composite statistical hypothesis testing procedures [2, 5] that employ the estimates of the characteristic quantities.

Damage localization. In the *baseline phase* the modeling of the structure via VFP-ARX models is achieved via standard identification procedures [4, 6] that involve a series of experiments performed either physically or via simulation, using, for instance, finite element models. Each experiment is characterized by a specific damage magnitude occurring at predetermined locations of a 3D Cartesian coordinate system with the complete series covering the combinations of all admissible damage magnitudes at all possible locations. Herein only a single damage magnitude (complete loosening of a bolt) is considered in the 3D space of a truss structure. For this reason a total number of $M_1 \times M_2 \times M_3$ experiments are performed. Each experiment is characterized by a specific damage location with coordinates k^x, k^y, k^z, along the x, y, z axis, respectively, with the complete series covering the required range of each variable, say $[k^x_{min},\ k^x_{max}]$, $[k^y_{min},\ k^y_{max}]$ and $[k^z_{min},\ k^z_{max}]$ via the discretizations $k^x = k^x_1, k^x_2, \ldots, k^x_{M_1}$, $k^y = k^y_1, k^y_2, \ldots, k^y_{M_2}$ and $k^z = k^z_1, k^z_2, \ldots, k^z_{M_3}$. For the identification of a model corresponding to a single damage magnitude located at any point in the three dimensional space of the structure, the following vector operating parameter $\boldsymbol{k}$ is defined in this study as:

$$\boldsymbol{k} = [k^x_i\ k^y_j\ k^z_h]^T \Longleftrightarrow k_{i,j,h},\ \ i = 1, \ldots, M_1,\ \ j = 1, \ldots, M_2,\ \ h = 1, \ldots, M_3 \quad (1)$$

with $k_{i,j,h}$ designating the state of the structure corresponding to damage location at i-th, j-th, h-th points on x, y, z axis, respectively. One more component representing the damage magnitude is added in vector $\boldsymbol{k}$ in case of multiple damage magnitudes.

This procedure yields a pool of excitation-response signal pairs (each of length N):

$$x_{\boldsymbol{k}}[t], y_{\boldsymbol{k}}[t] \quad \text{with} \ \ t = 1, \ldots, N, \quad (2)$$
$$k^x \in \{k^x_1, \ldots, k^x_{M_1}\},\ k^y \in \{k^y_1, \ldots, k^y_{M_2}\}, k^z \in \{k^z_1, \ldots, k^z_{M_3}\}.$$

Then a mathematical description of the structure for the considered damage in the three axis of the Cartesian system, is obtained in the form of a VFP-ARX model. In the case of several vibration measurement locations, an array of such models (or vector model) may be obtained, with each model corresponding to each measurement location.

The VFP-ARX(na, nb) model structure postulated for treating the problem is of the form [4]:

$$y_{\boldsymbol{k}}[t] + \sum_{i=1}^{na} a_i(\boldsymbol{k}) \cdot y_{\boldsymbol{k}}[t-i] = \sum_{i=0}^{nb} b_i(\boldsymbol{k}) \cdot x_{\boldsymbol{k}}[t-i] + e_{\boldsymbol{k}}[t] \quad (3)$$

$$e_{\boldsymbol{k}}[t] \sim \ \text{iid}\,\mathcal{N}\big(0, \sigma_e^2(\boldsymbol{k})\big) \quad \boldsymbol{k} \in \mathbb{R}^3 \quad (4)$$

$$a_i(\boldsymbol{k}) \triangleq \sum_{j=1}^{p} a_{i,j} \cdot G_j(\boldsymbol{k}), \quad b_i(\boldsymbol{k}) \triangleq \sum_{j=1}^{p} b_{i,j} \cdot G_j(\boldsymbol{k}) \quad (5)$$

with na, nb designating the AutoRegressive (AR) and eXogenous (X) orders, respectively, $x_{\boldsymbol{k}}[t]$, $y_{\boldsymbol{k}}[t]$ the excitation and response signals, respectively, and $e_{\boldsymbol{k}}[t]$ the disturbance (innovations) signal that is a white (serially uncorrelated) zero-mean with variance $\sigma_e^2(\boldsymbol{k})$ and potentially cross-correlated with its counterparts corresponding to different experiments.

As Eq. (5) indicates, the AR and X parameters $a_i(\boldsymbol{k})$, $b_i(\boldsymbol{k})$ are modeled as explicit functions of the vector $\boldsymbol{k}$ belonging to a p-dimensional functional subspace spanned by the (mutually independent) functions $G_1(\boldsymbol{k}), G_2(\boldsymbol{k}), \ldots, G_p(\boldsymbol{k})$ (*functional basis*). The functional basis consist of polynomials of three variables (vector polynomials) obtained as tensor products from univariate polynomials (of the Chebyshev, Legendre, Jacobi and other families). The constants $a_{i,j}$, $b_{i,j}$ designate the AR and X, respectively, coefficients of projection.

The VFP-ARX model, corresponding to all operating parameters $\boldsymbol{k}(k_{1,1,1}, k_{1,1,2}, \ldots, k_{M_1,M_2,M_3})$ considered in the experiments, can be written in a linear regression form as:

$$\boldsymbol{y} = \boldsymbol{\Phi} \cdot \boldsymbol{\theta} + \boldsymbol{e} \tag{6}$$

where $\boldsymbol{\Phi}$ is the regressor matrix, $\boldsymbol{\theta}$ the parameter vector and $\boldsymbol{e}$ the model's residuals. The projection coefficient vector $\boldsymbol{\theta}$ may be estimated based on the Ordinary Least Squares (OLS) estimator [6]:

$$\hat{\boldsymbol{\theta}}_{\text{OLS}} = \left[\boldsymbol{\Phi}^T \boldsymbol{\Phi}\right]^{-1} \cdot \left[\boldsymbol{\Phi}^T \boldsymbol{y}\right] \tag{7}$$

In the *inspection phase*, a pair of excitation $x[t]$ and response $y[t]$ signals are obtained from a current (unknown) state of the structure. Damage localization is based on the re-parameterized VFP-ARX model, in terms of $\boldsymbol{k}$ and $\sigma_e^2(\boldsymbol{k})$ with the projection coefficients to be available from the baseline phase:

$$M\big(\boldsymbol{k}, \sigma_e^2(\boldsymbol{k})\big): \quad y[t] + \sum_{i=1}^{na} a_i(\boldsymbol{k}) \cdot y[t-i] = \sum_{i=0}^{nb} b_i(\boldsymbol{k}) \cdot x[t-i] + e[t]. \tag{8}$$

The current excitation and response signals are driven through the VFP-ARX(na, nb) model estimated in the baseline phase and the currently unknown operating parameter vector $\boldsymbol{k}$ is estimated. This may be achieved via the following Nonlinear Least Squares (NLS) and variance estimators (refer to [6, pp. 327-329] for details on NLS estimation):

$$\widehat{\boldsymbol{k}} = \arg\min_{\boldsymbol{k}} \sum_{t=1}^{N} e^2[t] \qquad \sigma_e^2(\widehat{\boldsymbol{k}}) = \frac{1}{N} \sum_{t=1}^{N} e^2[t, \widehat{\boldsymbol{k}}] \tag{9}$$

the first one realized via constrained nonlinear optimization (Sequential Quadratic Programming – SQP; *matlab function: fmincon.m*).

The first estimator may be shown [4] to be asymptotically Gaussian distributed, with mean equal to the true $\boldsymbol{k}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{k}}$ $\big(\widehat{\boldsymbol{k}} \sim \mathcal{N}(\boldsymbol{k}, \boldsymbol{\Sigma}_{\boldsymbol{k}})\big)$ coinciding with the Cramer-Rao lower bound.

Damage localization is herein based on the interval estimates of k^x, k^y and k^z which are constructed based on the $\widehat{\boldsymbol{k}}$, $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{k}}$ estimates obtained from the re-parameterized VFP-ARX model. The interval estimates of k^x, k^y and k^z at the α risk level, which are constructed as in [4], are:

$$k^i \quad \text{interval estimate:} \qquad \left[\widehat{k}^i + t_{\frac{\alpha}{2}}(N-1) \cdot \widehat{\sigma}_{k^i},\ \widehat{k}^i + t_{1-\frac{\alpha}{2}}(N-1) \cdot \widehat{\sigma}_{k^i}\right] \tag{10}$$

with $i \in \{x, y, z\}$ and $\widehat{\sigma}_{k^i}$ is the positive square root of the diagonal element of $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{k}}$, which for $i = x$ corresponds to the first diagonal element, for $i = y$ to the second diagonal element and for $i = z$ to the third diagonal element.

TABLE II. DAMAGE DETECTION AND LOCALIZATION DETAILS

Damage Detection			
Technique PSD/FRF based	Segment length 1020 samples	No of Segments 10	Window type Hamming
Technique SPRT based	Estimated model ARX(76,76)	Dimension of $\boldsymbol{\theta}$ 153	σ_o/σ_1 1.1
Damage Localization			
Method FMBM	Estimated model VFP-ARX(76,76)	Dimension of $\boldsymbol{\theta}$ 2448	$M_1 \times M_2 \times M_3$ 16

Trivariate confidence regions for $\boldsymbol{k} = [k^x\ k^y\ k^z]^T$ may be obtained by observing that the quantity $(\widehat{\boldsymbol{k}} - \boldsymbol{k})^T \boldsymbol{\Sigma_k}^{-1} (\widehat{\boldsymbol{k}} - \boldsymbol{k}) \sim \chi^2_{1-\alpha,3}$, follows chi-square distribution with three degrees of freedom [6, p. 558]. Thus the probability that:

$$(\widehat{\boldsymbol{k}} - \boldsymbol{k})^T \boldsymbol{\Sigma_k}^{-1} (\widehat{\boldsymbol{k}} - \boldsymbol{k}) \leq \chi^2_{1-\alpha,3} \tag{11}$$

is equal to $1-\alpha$ ($\chi^2_{1-\alpha,3}$ designating the χ^2 distribution's with three degrees of freedom $1-\alpha$ critical point). This expression defines a three dimensional ellipsoid on the (k^x, k^y, k^z) space within which the true damage coordinates (k^x, k^y, k^z) should lie with probability $(1-\alpha)$, or equivalently, with risk α *(trivariate confidence regions)*. The shape of the ellipsoid is determined by $\boldsymbol{\Sigma_k}$ which in practice is replaced by its estimate.

DAMAGE DETECTION AND LOCALIZATION RESULTS

Damage detection. In the *baseline phase* the characteristic quantities for the non-parametric PSD and FRF based techniques are estimated by using measurement signals of the healthy structure *(Matlab functions: pwelch.m, tfestimate.m)*. Also, the parametric SPRT based technique employs an ARX(76,76) model *(Matlab function: arx.m)* which is estimated based on a single pair of excitation-response signals of the healthy structure. A second pair of signals from the healthy structure is driven through the estimated ARX(76,76) and the nominal residual standard deviation σ_o is computed. Furthermore the user defined residual standard deviation ratio σ_o/σ_1 is selected equal to 1.1, designating a 10% increase in the nominal standard deviation (see [5] for details). All details are shown in Table II.

In the *inspection phase* damage detection is initially based on signals from the healthy structure where all techniques found to identify the healthy condition of the structure without false alarms. The techniques' effectiveness is in the following assessed based on the acquired signals from the damaged structure. No missed damage situations occur except for one case where the PSD based technique has failed to effectively detect damage. The damage detection results for all techniques and the corresponding risk levels are presented in Table III.

Damage localization. In the *baseline phase* a VFP-ARX(76,76) model is estimated based on the Set A (Table I) of experiments where each acquired pair of force-acceleration signals corresponds to a certain damage location in each of the 16 nodes of the truss structure [see Figure 1(b)]. The VFP-ARX modeling procedure leads to

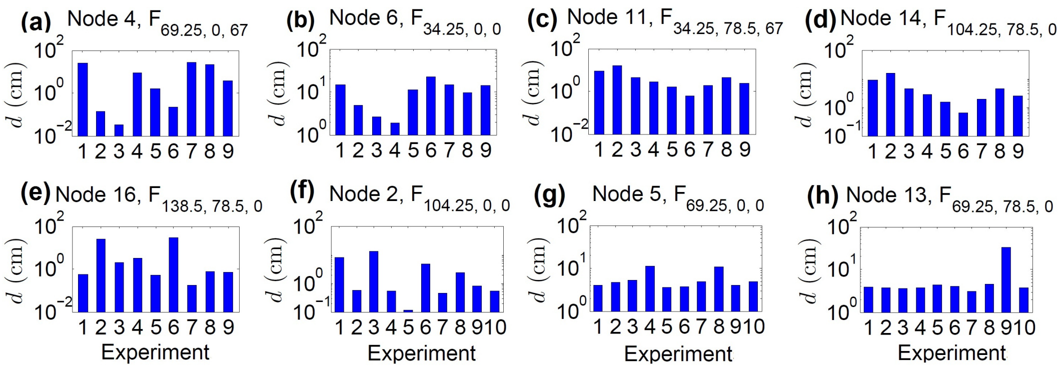

Figure 2. Indicative damage localization results in terms of the Euclidean distance between the true $\boldsymbol{k}$ and its estimate $\widehat{\boldsymbol{k}}$ for 75 test cases. (a)-(e) Damage cases from Set B of experiments. (f)-(h) Damage cases from Set C of experiments.

TABLE III. DAMAGE DETECTION RESULTS

Method	Risk Level	False Alarms	Missed Damages
PSD	$\alpha = 2 \cdot 10^{-4}$	$0/9$	$1/74$
FRF	$\alpha = 2 \cdot 10^{-14}$	$0/9$	$0/74$
SPRT	$\alpha = \beta = 1 \cdot 10^{-2}$	$0/8$	$0/74$

a functional subspace consisting of $p = 16$ trivariate Shifted Legendre polynomials (see details in Table II). Thus the identified VFP-ARX(76,76) model may represent the damaged truss structure with damages located at any point in the three dimensional space covering the following ranges of coordinates: $k^x \in [0, 138.25]$ cm, $k^y \in [0, 78.5]$ cm, $k^z \in [0, 67]$ cm [see also Figure 1(b)].

In the *inspection phase* the assessment of the proposed damage localization method is initially based on the Set B of experiments with damages at the same locations with those used in VFP-ARX(76,76) model identification. Set C with damages in different locations is finally used for method's assessment.

The method's effectiveness is demonstrated in terms of the Euclidean distance between the true $\boldsymbol{k}$ and the estimated $\widehat{\boldsymbol{k}}$ damage locations. Indicative damage localization results from 75 experiments are shown in Figure 2. These include 9 series of experiments at 5 damage locations from Set B and all experiments from Set C. As it is shown the method achieves very good damage localization. In 93.7% of the cases the estimated damage location is closer than 25 cm to the true location.

Furthermore the method's effectiveness is demonstrated in terms of damage location point estimates along with their trivariate confidence regions (three dimensional ellipsoids). Indicative results for eight test cases where again the effective damage localization is evident are depicted in Figure 3. In certain cases the true damage location is outside (but very close) to the confidence regions.

CONCLUDING REMARKS

The goal of this study was the extension of a recently introduced statistical time series method for both damage detection and precise localization in three dimensional continuous structural topologies. The proposed extension achieved with the postulation of VFP-ARX models with three component operating parameter vector, each one corresponding to a dimension of a 3D Cartesian coordinate system. Damage

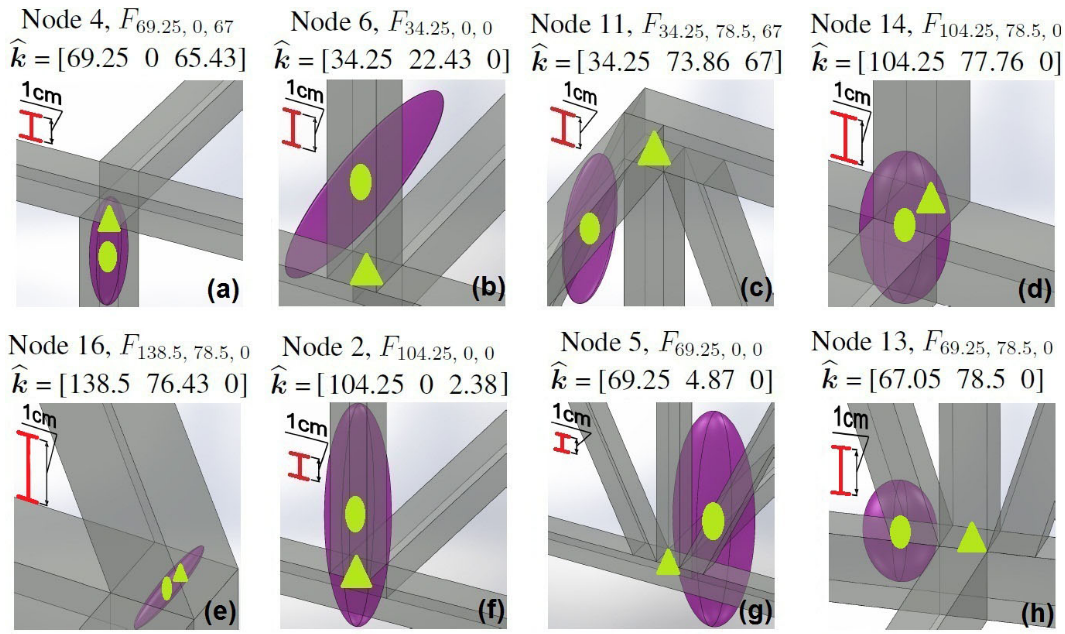

Figure 3. Indicative damage localization results in terms of point estimates and trivariate confidence regions (ellipsoids) for 8 test cases (△: true damage location, ○: point estimate). The coordinates of the true and estimated damage location are shown above each plot.

detection was achieved based on a scheme of three techniques and simple hypothesis testing procedures while damage localization was based on the proposed VFP-ARX model. A number of test cases with the healthy and damaged truss are used and the achieved results are very promising suggesting effective damage detection (complete loosening of joint bolts) in all considered test cases while damage localization was judged as very good in terms of the Euclidean distance between the true and the estimated locations.

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