Fault Detection and Identification in Time-Varying Structures via an FS-TAR Model Based Method: Application to a Pick-and-Place Mechanism

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ABSTRACT

The problem of vibration-based Fault Detection and Identification (FDI) in inherently Time-Varying (TV) structures is tackled via a statistical time series type method. This method is based on Functional Series Time-dependent AutoRegressive (FS-TAR) models combined with an appropriate statistical decision making scheme. Its performance is experimentally assessed via its application to fault detection and identification of a pick-and-place mechanism. The faults considered are of various types and occurrence locations, while their diagnosis is based solely on a single non-stationary vibration response signal acquired during normal operation. The method is shown to achieve effective FDI for all fault scenarios considered.

INTRODUCTION

Structures characterized by properties that vary with time are referred as *Time-Varying (TV)*, or else *non-stationary*. Such structures are widely used in a number of applications and include robotic structures and mechanisms used in assembly lines, flexible mechanisms and variable geometry structures used in aerospace technology, rotating machinery, and so on. In a large number of such cases, the structures operate continuously, and their in-operation inspection based on automated decision making is of critical importance.

In recent years, significant attention has been paid to structural Fault Detection and Identification (FDI) via vibration based statistical time series methods [1, 2]. These utilize random excitation and/or vibration response signals (time series), along with statistical model building and decision making tools, for inferring the health state of a structure. They offer a number of advantages over alternative FDI methods such as no requirement for physics based or finite element models, no need to interrupt normal operation, and effective treatment of uncertainties [2].

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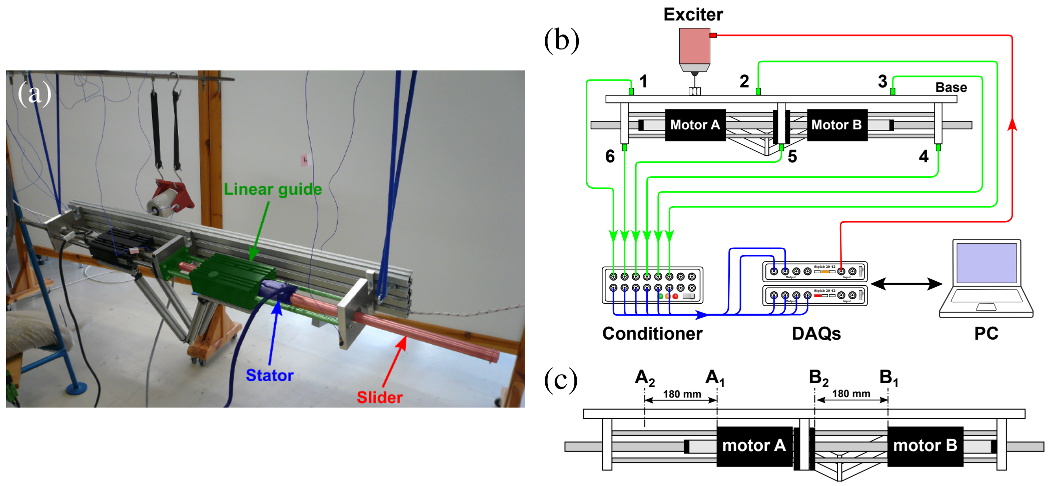

Figure 1: The pick-and-place mechanism and the experimental setup: (a) photo, (b) schematic diagram, and (c) the motor end positions.

The aforementioned advantages make statistical time series FDI methods particularly attractive for TV structures as well. The development of such a method is on the focus of the present study. The method introduced employs a single random vibration response signal from the structure in its healthy state, as well as from a number of potential faulty states, obtained during a single operational cycle. For each structural state a suitable non-stationary Functional Series Time-Dependent AutoRegressive (FS-TAR) model is identified that models the corresponding TV dynamics. An appropriate statistical quantity (characteristic quantity) is subsequently extracted which characterizes the structural state in each case (baseline phase). Fault detection and identification is then accomplished via statistical decision making consisting of comparing, in a statistical sense, the current characteristic quantity with that of each potential state as pre-determined in the baseline phase (inspection phase).

The effectiveness of the method is assessed via its application to a TV pick-and-place mechanism consisting of two linear motors which follow prescribed motion profiles. A number of test cases (experiments) are considered, each one corresponding to a specific fault scenario (loosening or removing one or more bolts from various parts of the mechanism or adding small mass on a motor).

THE MECHANISM, THE FAULTS & THE EXPERIMENTAL SET-UP

The structural system employed in the study is a 2-DOF pick-and-place mechanism consisting of two coaxially aligned linear motors (LinMot P01-37$\times$120) that carry prismatic links connected to their ends, with the whole mechanism being clamped on an aluminium base (Fig. 1(a)). The mechanism is suspended through two bungee cords from a long rigid beam sustained by two heavy-type stands. The excitation is a zero-mean Gaussian random stationary force which is vertically (with respect to the base) exerted via an electromechanical shaker (LDS V201) equipped with a stinger (Figs. 1(a) and (b)). The vertical (with respect to the base) vibration of the mechanism is measured at six selected locations (locations 1-6; Fig. 1(b)) via lightweight piezoelectric accelerometers. The FDI results of the present study are

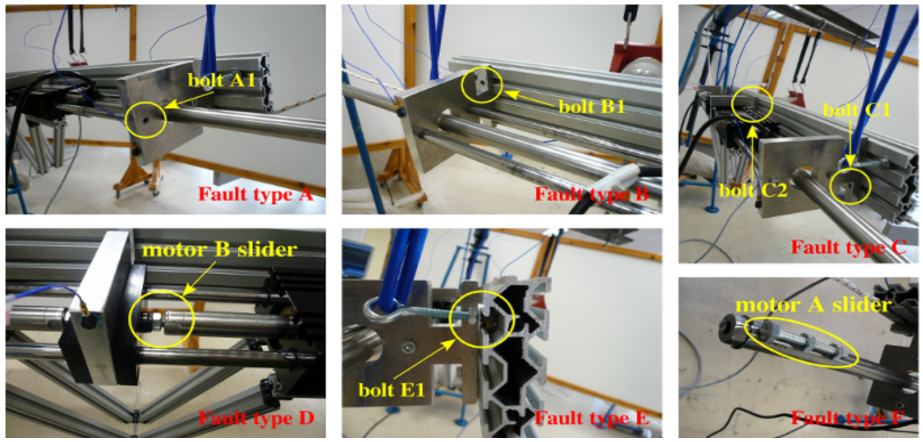

Figure 2: The considered fault types (A, B, C, D, E and F).

Table 1: The fault types, numbers of FDI experiments and signal details.

Structural State	Description	FDI experiments
Healthy	—	40
Fault Type A	removal of bolt A1	40
Fault Type B	removal of bolt B1	40
Fault Type C	removal of bolts C1 and C2	40
Fault Type D	loosening of motor B slider	40
Fault Type E	loosening of bolt E1	40
Fault Type F	adding a mass on motor A slider	40
Analysis bandwidth:	5 – 200 Hz	
Sampling frequency:	$f_s = 512$ Hz	
Signal length:	$N = 5{,}120$ samples ($= 10$ s)	

based on the non-stationary vibration response measured at location 4 (Fig. 1(b)).

The fault scenarios considered correspond to the loosening or removal of various bolts at different points of the mechanism, loosening the slider of motor B, and adding a mass at the free end of the slider of motor A (Fig. 2). In total, six distinct fault scenarios (types) are considered and are summarized in Table 1. The assessment of the statistical time series method with respect to the fault detection and identification subproblems is based on 40 experiments for the healthy and 40 experiments for each considered fault state of the mechanism (fault types $A, B, \ldots, F$ – see Table 1). However, an additional experiment with the mechanism under its healthy state and one for each faulty state (fault types $A, B, \ldots, F$) are executed and used in the baseline phase.

During a single experiment the linear motors move from their rightmost to their leftmost position and back (Fig. 1(c); from A_1–B_1 to A_2–B_2 and back to A_1–B_1) following a sinusoidal reference position profile lasting 10 s. The vibration acceleration signals are sampled at f_s = 512 Hz, each one being 10 s (N = 5,120 samples) long. The frequency range of interest is selected as 5 – 200 Hz (Table 1).

THE FAULT DETECTION & IDENTIFICATION METHOD

The FDI method consists of two phases: (a) The baseline phase, which includes the modelling of the TV dynamics of the mechanism based on measured response signals and the extraction of an appropriate statistical quantity Q characterizing the structural state in each case, and (b) the inspection phase that is performed periodically or on demand during the mechanism's service cycle. This second phase utilizes the current vibration response signal – with the mechanism being under its current (unknown) state – and performs FDI through statistical decision making tests that compare the current characteristic quantity Q_u with that of each potential state as determined in the baseline phase (Q_o for the healthy state, or $Q_A, \dots, Q_F$ for the faulty states).

Baseline Phase: Modelling of the TV Dynamics

An FS-TAR$(n_a)_{[p_a,p_s]}$ model, with n_a denoting its AutoRegressive (AR) order, p_a and p_s the AR and innovations standard deviation (std) functional basis dimensionalities, respectively, is of the form:

$$x[t] + \sum_{i=1}^{n_a} a_i[t] \cdot x[t-i] = e[t], \qquad e[t] \sim \text{NID}(0, \sigma_e^2[t]), \qquad t = 1, \dots, N \quad (1)$$

with t designating normalized discrete time, $x[t]$ the non-stationary response signal, and $e[t]$ the corresponding innovations (residual) sequence which is characterized by zero-mean and time-dependent std $\sigma_e[t]$. NID$(\cdot)$ stands for Normally Independently Distributed with the indicated mean and variance.

The model parameter $a_i[t]$, along with the innovations time-dependent std $\sigma_e[t]$, are considered to belong to specific functional subspaces and may be thus expanded on respective basis functions as:

$$a_i[t] \triangleq \sum_{j=1}^{p_a} a_{i,j} \cdot G_{b_a(j)}[t], \qquad \sigma_e[t] \triangleq \sum_{j=1}^{p_s} s_j \cdot G_{b_s(j)}[t] \quad (2)$$

where $G_j[t]$ stands for the j-th basis function and $b_a(j)(j = 1, ..., p_a)$ and $b_s(j)(j = 1, ..., p_s)$ designate the functions included in each basis. An FS-TAR model is thus parameterized in terms of its time-invariant projection coefficients $a_{i,j}, s_j$, while a specific model structure defined by the AR model order n_a and the functional subspaces $\mathcal{F}_{AR} \triangleq \{G_{b_a(1)}[t], \ ..., \ G_{b_a(p_a)}[t]\}$ and $\mathcal{F}_{\sigma_e} \triangleq \{G_{b_s(1)}[t], \ ..., \ G_{b_s(p_s)}[t]\}$.

Model parameter estimation.

The estimation of the projection coefficient vector $\boldsymbol{\theta} = [\boldsymbol{a}^T, \boldsymbol{s}^T]^T$, consisting of the AR and innovations std coefficient of projection vectors $\boldsymbol{a} \triangleq [a_{1,1} \dots a_{n_a,p_a}]^T$ and $\boldsymbol{s} \triangleq [s_1 \dots s_{p_s}]^T$, respectively, is based on available non-stationary vibration response signals and a selected specific model structure.

In the present study the estimation of $\boldsymbol{\theta}$ is based on the Multi-Stage (MS) method [4] which is very briefly presented below.

Stage 1. Initial AR projection coefficient vector estimation via the Ordinary Least Squares (OLS) estimator:

$$\widehat{\boldsymbol{a}}^{\text{OLS}} = \left[\frac{1}{N} \sum_{t_a+1}^{N} \boldsymbol{\phi}[t] \cdot \boldsymbol{\phi}^T[t] \right]^{-1} \cdot \left[\frac{1}{N} \sum_{t_a+1}^{N} \boldsymbol{\phi}[t] \cdot x[t] \right] \quad (3)$$

where $\boldsymbol{\phi}[t] \triangleq [\ -G_{b_a(1)}[t] \cdot x[t-1], \dots, -G_{b_a(p_a)}[t] \cdot x[t-n_a]\ ]^T$.

Stage 2. Innovations std projection coefficient vector $\boldsymbol{s}$ estimation by maximizing the log-likelihood of the FS-TAR model [3] with respect to $\boldsymbol{s}$:

$$\widehat{\boldsymbol{s}}^{\text{ML}} = \arg\min_{\boldsymbol{s}} \frac{1}{N} \sum_{t_a+1}^{N} \left[\ln\left(\left(\boldsymbol{g}_s^T[t] \cdot \boldsymbol{s} \right)^2 \right) + \frac{e^2[t, \widehat{\boldsymbol{a}}^{\text{OLS}}]}{\left(\boldsymbol{g}_s^T[t] \cdot \boldsymbol{s} \right)^2} \right] \tag{4}$$

where $\boldsymbol{g}_s[t] \triangleq \left[G_{b_s(1)}[t] \quad G_{b_s(2)}[t] \ \ldots \ G_{b_s(p_s)}[t] \right]^T$. The prediction errors $e[t, \widehat{\boldsymbol{a}}^{\text{OLS}}]$ are obtained by the relation $e[t, \widehat{\boldsymbol{a}}^{\text{OLS}}] = x[t] - \boldsymbol{\phi}^T[t] \cdot \widehat{\boldsymbol{a}}^{\text{OLS}}$. Estimation of $\boldsymbol{s}$ based on Eq. (4) constitutes a nonlinear optimization problem, usually of low dimensionality p_s, that may be tackled via iterative optimization techniques.

Stage 3. Final AR projection coefficient vector estimation via a Weighted Least Squares (WLS) estimator:

$$\widehat{\boldsymbol{a}} = \left[\frac{1}{N} \sum_{t_a+1}^{N} \frac{\boldsymbol{\phi}[t] \cdot \boldsymbol{\phi}^T[t]}{\left(\boldsymbol{g}_s^T[t] \cdot \widehat{\boldsymbol{s}}^{\text{ML}} \right)^2} \right]^{-1} \cdot \left[\frac{1}{N} \sum_{t_a+1}^{N} \frac{\boldsymbol{\phi}[t] \cdot x[t]}{\left(\boldsymbol{g}_s^T[t] \cdot \widehat{\boldsymbol{s}}^{\text{ML}} \right)^2} \right] \tag{5}$$

Remark. Stages 2 and 3 are iterated until convergence criteria with respect to the estimated parameters and the value of a suitable prediction error function (presently the Residual Sum of Squares RSS $= \sum_{t=1}^{N} e^2[t, \widehat{\boldsymbol{a}}]$), are achieved.

Model structure estimation.

Given a basis function family, model structure estimation refers to the estimation of the set of integers $\mathcal{M} = \{n_a, p_a, p_s, b_a(j), b_s(j)\}$. This is tackled via a two-phase procedure based on backward regression and the minimization of the Bayesian Information Criterion (BIC) [3].

Inspection Phase: Fault Detection and Identification

The characteristic quantity Q used for fault detection and identification is the AR coefficients of projection vector $\boldsymbol{a}$. For sufficiently long signals, the $\boldsymbol{a}$ MS estimator is (under mild conditions) Gaussian distributed with mean equal to its true value, say $\mathring{\boldsymbol{a}}$, and covariance matrix $\boldsymbol{P}$, thus $\boldsymbol{a} \sim \mathcal{N}(\mathring{\boldsymbol{a}}, \boldsymbol{P})$ [4].

Fault detection is based on testing for statistically significant changes in the parameter vector $\boldsymbol{a}$ between the nominal and current state of the TV mechanism through the hypothesis testing problem [2]:

$$\begin{array}{lll} H_o & : & \delta\boldsymbol{a} = \boldsymbol{a}_o - \boldsymbol{a}_u = 0 \qquad \text{(null hypothesis – healthy mechanism)} \\ H_1 & : & \delta\boldsymbol{a} = \boldsymbol{a}_o - \boldsymbol{a}_u \neq 0 \qquad \text{(alternative hypothesis – faulty mechanism)} \end{array} \tag{6}$$

where $\boldsymbol{a}_o$ and $\boldsymbol{a}_u$ designate the true coefficient of projection vector for the healthy and current (being in unknown state), respectively, structural system model. Based on the normality of the AR parameter vector estimator, the following statistical test based on the χ^2 distribution may be constructed at the α (type I) risk level (false alarm probability) [2]:

$$\begin{array}{lll} \chi^2_{\boldsymbol{a}} = \delta\widehat{\boldsymbol{a}}^T \cdot \delta\boldsymbol{P}^{-1} \cdot \delta\widehat{\boldsymbol{a}} \leq \chi^2_{1-\alpha}(n_a p_a) & \Longrightarrow & H_0 \text{ is accepted (healthy mechanism)} \\ \text{Else} & \Longrightarrow & H_1 \text{ is accepted (faulty mechanism)} \end{array} \tag{7}$$

with $\delta\widehat{\boldsymbol{a}} = \widehat{\boldsymbol{a}}_o - \widehat{\boldsymbol{a}}_u$, $\delta\boldsymbol{P} = 2\widehat{\boldsymbol{P}}_o$, and $\chi^2_{1-\alpha}(n_a p_a)$ designating the $1-\alpha$ critical point of the χ^2 distribution with $n_a p_a$ (parameter vector $\boldsymbol{a}$ dimensionality) degrees of freedom.

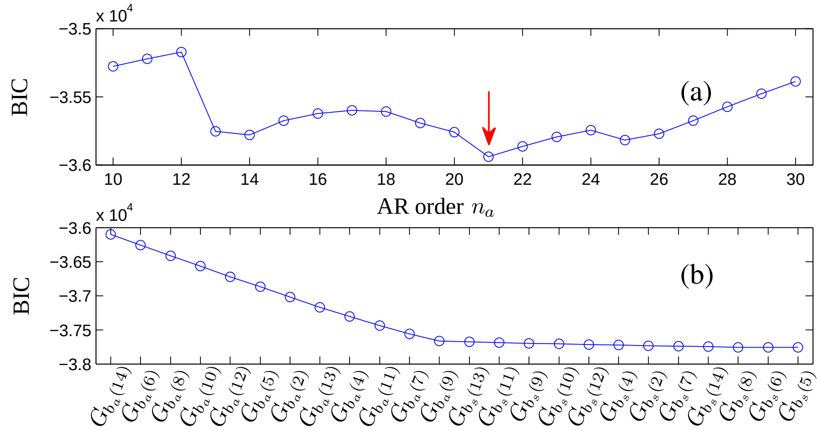

Figure 3: Healthy baseline FS-TAR model structure selection: (a) AR order selection based on the BIC using an extended and complete functional subspace ($p_a = p_s = 15$) – the selected order is indicated by an arrow; (b) BIC values obtained by sequentially dropping the indicated basis functions until no further reduction is possible (backward regression).

Table 2: Parameter estimation method implementation details.

OLS	MATLAB *mldivide* function (Householder transformation)
$\boldsymbol{a}$ MS estimation	termination rules: $\frac{\mid \text{RSS}_i - \text{RSS}_{i-1} \mid}{1+ \mid \text{RSS}_{i-1} \mid} < 10^{-8}$ and $\frac{\parallel \boldsymbol{a}_i - \boldsymbol{a}_{i-1} \parallel}{1+ \parallel \boldsymbol{a}_{i-1} \parallel} < 10^{-8}$
$\boldsymbol{s}$ ML nonlinear optimization	MATLAB *fminsearch* function (Nelder-Mead simplex method), termination rules: $\frac{\mid (\ln \mathcal{L})_i - (\ln \mathcal{L})_{i-1} \mid}{1+ \mid (\ln \mathcal{L})_{i-1} \mid} < 10^{-8}$ and $\frac{\parallel \boldsymbol{s}_i - \boldsymbol{s}_{i-1} \parallel}{1+ \parallel \boldsymbol{s}_{i-1} \parallel} < 10^{-12}$

$\ln \mathcal{L}$: log-likelihood function

Fault identification is similarly based on proper comparison of the parameter vector $\boldsymbol{a}_u$ belonging to the current state of the mechanism to each one of $\boldsymbol{a}_A, \boldsymbol{a}_B, \ldots,$ $\boldsymbol{a}_F$ corresponding to different fault types via statistical hypothesis testing.

FAULT DETECTION & IDENTIFICATION RESULTS

Baseline Phase

FS-TAR modelling of the TV mechanism under the various structural states is presently considered. A functional basis spanned by the discrete Fourier transform functions:

$$G_0[t] = 1, \;\; G_{2\kappa-1}[t] = \cos[2\kappa\pi(t-1)/(N-1)], \;\; G_{2\kappa}[t] = \sin[2\kappa\,\pi\,(t-1)/(N-1)],$$

with $\kappa = 1, 2, \ldots$, and $t = 1, \ldots, N$ is adopted, motivated by the periodic nature of the motion profile. At first an extended and complete functional subspace of dimensionality 15 is employed and FS-TAR$(n_a)_{[15,15]}$ models with $n_a = 10, \ldots, 30$ are estimated with the AR order being selected based on the BIC (Fig. 3(a)). The initial functional subspace is subsequently reduced based on the BIC and a backward regression procedure (Fig. 3(b)). The implementation details for the parameter estimation method are summarized in Table 2, while the model structure details for all obtained models are summarized in Table 3.

The estimated, based on the healthy baseline model, dynamics are shown in Fig. 4, where the "frozen-time" FS-TAR$(21)_{[3,3]}$ based PSD estimate is contrasted to the

Table 3: Estimated baseline FS-TAR models: model structure details.

Structural State	n_a	b_a	p_a	b_s	p_s
Healthy	21	[0 1 3]	3	[0 1 3]	3
Fault Type A	22	[0 1 3]	3	[0 1 3]	3
Fault Type B	22	[0 1 3]	3	[0 3 5]	3
Fault Type C	21	[0 1 3]	3	[0 1 3 5 9 11 12]	7
Fault Type D	22	[0 1 3]	3	[0 1 3]	3
Fault Type E	21	[0 1 3]	3	[0 1 3]	3
Fault Type F	22	[0 1 3]	3	[0 3]	2

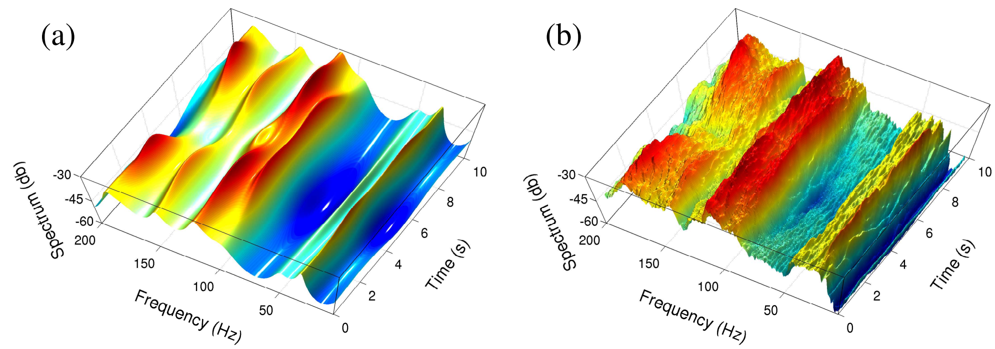

Figure 4: The estimated healthy TV structural dynamics expressed via TV PSD estimates: (a) FS-TAR based "frozen-time" TV PSD estimate, and (b) sample mean spectrogram obtained from 41 non-stationary experiments.

mean value of the spectrograms obtained from the 41 non-stationary experiments with the mechanism being under in its healthy state. The agreement between the FS-TAR based estimate and the mean spectrogram is evident, although the former is more clear and informative.

Inspection Phase

Fault detection and identification is based on the vibration response of a single accelerometer (Fig. 1 - location 4) along with the FS-TAR$(21)_{[3,3]}$ model identified in the baseline phase. In addition, corresponding FS-TAR models are identified in each test case using the current response signal (inspection phase).

Figure 5 presents the fault detection results. The test statistics corresponding to the healthy mechanism are shown in circles (40 experiments), while the test statistics corresponding to the various fault types are presented with symbols of different colour (different for each fault type; 40 experiments per fault type). Evidently, correct detection is obtained in each test case, as the test statistic is shown not to exceed the critical limit (at the $\alpha = 10^{-8}$ risk level) in all healthy cases, while it exceeds it in all faulty cases (note the logarithmic scale on the vertical axis in Fig. 5).

Fault identification is similarly based on the mechanism vibration response of accelerometer at location 4 and the baseline FS-TAR models identified for each faulty case. The fault identification results for all 240 test cases (each related to a particular faulty state) are summarized in Table 4 and indicate correct identification for fault types A, B, C and D, while a small number of fault missclassifications are obtained for fault cases E and F.

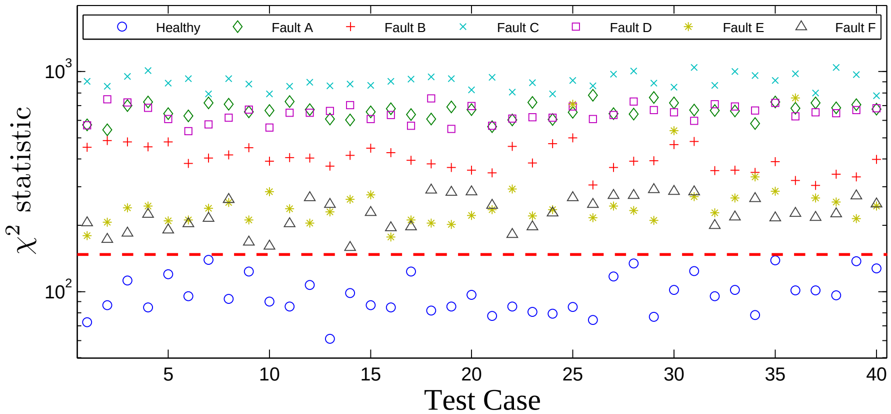

Figure 5: Fault detection results for all 280 experiments: A fault is detected if the test statistic exceeds the critical limit (— — —; risk level $\alpha = 10^{-8}$).

Table 4: Fault detection and identification summary results.

Fault Detection						
False Alarms	Missed Faults					
Healthy	Fault A	Fault B	Fault C	Fault D	Fault E	Fault F
0/40	0/40	0/40	0/40	0/40	0/40	0/40
Fault Identification (misclassifications)						
—	Fault A	Fault B	Fault C	Fault D	Fault E	Fault F
—	0/240	0/240	0/240	0/240	16/240	18/240

CONCLUSIONS

A statistical time series vibration based method for FDI in TV structures was introduced. The method is based on non-stationary FS-TAR models and a proper statistical decision making scheme utilizing a single vibration response signal acquired from the structure during its normal operation. The method's effectiveness was assessed via its application to the problem of fault detection and identification for a pick-and-place mechanism consisting of two linear motors that follow prescribed motion profiles. The considered faults were successfully detected with no false alarms or missed faults, while correct identification was also achieved in most test cases.

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