A New Technique for Acoustic Source Localization in an Anisotropic Plate Without Knowing Its Material Properties

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ABSTRACT

The conventional triangulation technique cannot locate the acoustic source in an anisotropic plate because this technique requires the wave speed to be independent of the propagation direction which is not the case for an anisotropic plate. All methods, proposed so far for source localization in anisotropic plates, require either the knowledge of the direction dependent velocity profile or a dense array of sensors. In this paper a technique is proposed to locate the acoustic source in large anisotropic plates with the help of only six sensors without knowing the direction dependent velocity profile in the plate. The proposed technique should work equally well for monitoring large isotropic and anisotropic plates. For an isotropic plate the number of sensors required for the acoustic source localization can be reduced to four.

INTRODUCTION

Ultrasonic transducers can be used in two modes - active and passive modes [1] for monitoring structural damage. For active monitoring acoustic actuators generate ultrasonic signals [2] and under passive monitoring the acoustic event itself such as the impact of a foreign object, fiber breakage, or matrix cracking in a composite plate act as the acoustic source [3-4]. Ultrasonic sensors are placed in critical areas of the structure to efficiently receive ultrasonic signals and monitor its condition [5-10].

This paper focuses on the passive monitoring technique to locate the acoustic source in a plate. For isotropic plates the point of impact can be located after detecting the acoustic emission signal (generated by the acoustic source) by at least three sensors and applying the triangulation technique. However, if the plate is anisotropic then the triangulation technique does not work. An alternative method was proposed by Kundu *et al.* [11-13]. Their method is based on the optimization technique – minimization of a non-linear objective function or error function. In principle that technique should work equally well for isotropic and anisotropic plates. However, it requires *a priori* knowledge of the direction dependent velocity profile in the plate.

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Another source localization technique was proposed by Salamone *et al.* [14] exploiting the directivity properties of MFC (microfiber composite) sensors. Using three such sensors arranged as rosettes the principal strain directions were obtained. The wave propagation direction can be predicted from such rosette arrangement when that direction coincides with the principal strain direction, as is the case for an isotropic plate. The acoustic source in an isotropic plate can be localized from the point of intersection of two wave propagation directions obtained from two rosettes made from six MFC sensors. However, this technique does not work for an anisotropic plate since the wave propagation direction (group velocity or energy propagation direction) does not coincide with the principal strain direction for the anisotropic plate.

Earlier attempts of locating the acoustic source in anisotropic plates required the measurement of two dominant pulses in a waveform whose speeds of propagation, c_1 and c_2 were known, and the receiving sensors were to be placed as a sensor-array - on the periphery of a circle or on two orthogonal lines [15]. Other restrictions of the earlier analyses are [16], (1) the order of the elastic symmetry of the solid is to be orthorhombic or higher, (2) the principal axes of the solid are to be known *a priori* and to be oriented along the coordinate axes of the specimen, (3) the sensors comprising the receiving array must be placed on principal planes of the material. The last constraint condition may not be satisfied for single-crystal specimens that have been cut in an arbitrary orientation. Although the first constraint condition is approximately satisfied for most engineering materials it is not necessarily true all the time. Even the widely used engineering materials such as the fiber reinforced composite solids may violate this condition. Note that although fiber reinforced composite solids are often assumed to be orthotropic or transversely isotropic materials the non-uniform distribution of fibers may not make *xz*, *yz* and/or *xy* planes to be planes of symmetry, as observed experimentally by Kundu *et al.* [12]. Readers are referred to Cowin and Mehrabadi [17] for definitions of different types of symmetry and principal planes of symmetry. The method proposed by Kundu *et al.* [11-13] for locating the point of impact works well for any type of anisotropy because it uses the experimentally obtained direction dependent Lamb wave velocity profile. On the other hand the method proposed by Castagnede *et al.* [16] is based on the quasi-longitudinal bulk wave speeds. Their method works well for thick structures but fails for thin plates when sensors are placed far away from the impact point because then the received signal is dominated by the Lamb wave modes while the contributions of the longitudinal and shear bulk waves are negligible.

Recently McLaskey *et al.* [18] proposed beamforming array technique for localizing acoustic source in large plate type structures using eight sensors. Their technique also requires the knowledge of the wave velocity in the plate. In principle this technique can be extended to anisotropic plates but will also require the knowledge of the velocity profile in the plate.

Need for a New Technique

Currently we have a good handle of source localization for isotropic plates. For example in this case the conventional triangulation technique with three sensors and rosette arrangement with six sensors [14] work well. Source in an isotropic plate can be also localized by the beamforming array technique [18]. However, source localization in an anisotropic plate is relatively immature and requires *a priori* knowledge of the direction dependent velocity profile. When the velocity profile is known then the source can be accurately localized [11-13]. However, since Lamb waves in a plate can propagate in multiple modes with different velocity and attenuation values, and the speed of a specific mode also changes with frequency because of the dispersive nature of the mode [19] it is difficult to obtain a unique velocity profile in the plate. The velocity profile changes as the impacting object and its striking velocity change, or the crack formation mechanism changes because the propagating wave mode and its frequency are controlled by the exciting source. Therefore, in real life applications when the acoustic source (the impacting object or the crack formation mechanism) is not known then a large array of sensors must be embedded in the entire structure or optimally placed in critical regions where acoustic sources are most likely to occur [6, 20]. The sensor closest to the acoustic source generates the strongest signal and thus the acoustic source can be approximately localized. The accuracy of such source localization mechanism depends on the grid size or spacing between two successive sensors. Although this technique is reliable it requires a large number of sensors and processing of huge amount of recorded data by all these sensors.

If the acoustic source in an anisotropic plate can be localized with only six sensors and without knowing the velocity profile in the plate then that will be a significant improvement over the currently available techniques.

Formulation

Three receiving sensors S_1, S_2 and S_3 are mounted on the plate as shown in Figure 1. If the coordinates of three receiving sensors S_1, S_2 and S_3 are (x_1, y_1), (x_2, y_2) and (x_3, y_3), respectively then $x_2 = x_1+d$, $x_3 = x_1$, $y_2 = y_1$ and $y_3 = y_1+d$. The coordinate of the acoustic source (A) is given by (x_A, y_A). Impact of a foreign object or a crack formation can act as the acoustic source at A. The distance d between the sensors should be much smaller than the distance D between the acoustic source A and any sensor S_i. Therefore, the inclination angle θ of line AS_1 (see Figure 1) should be same for lines AS_2 and AS_3. Because of this assumption the received signals at these three sensors will be almost identical but slightly time shifted and the wave velocity in the direction from A to S_1, S_2 or S_3 should be almost same even for an anisotropic plate. Angle θ can be expressed as,

$$\theta = \tan^{-1}\left(\frac{y_1 - y_A}{x_1 - x_A}\right) \approx \tan^{-1}\left(\frac{y_2 - y_A}{x_2 - x_A}\right) \approx \tan^{-1}\left(\frac{y_3 - y_A}{x_3 - x_A}\right) \qquad (1)$$

After arriving at sensor S_1 the time taken by the wave front to reach sensors S_2 and S_3 can be denoted as Δt_{12} and Δt_{13}, respectively. These two time delays are given by,

$$\Delta t_{12} = \frac{d\cos\theta}{c(\theta)} \qquad (2)$$

$$\Delta t_{13} = \frac{d\sin\theta}{c(\theta)} \qquad (3)$$

Where $c(\theta)$ is the wave velocity in the θ direction. From Eqs. (2) and (3) one can easily obtain,

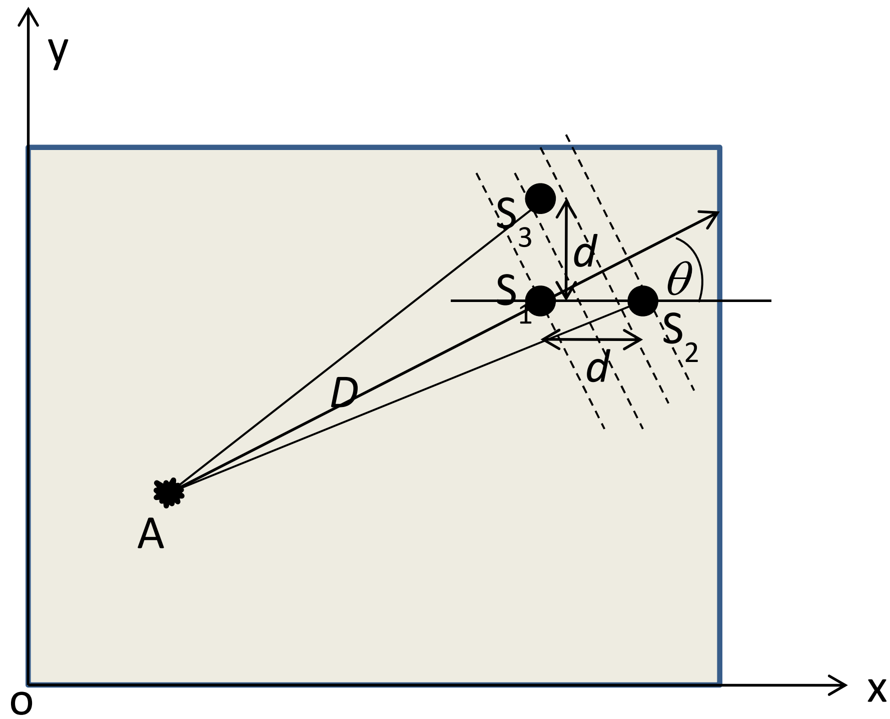

Figure 1. Acoustic source A and three sensors shown on a plate.

$$\theta = \tan^{-1}\left(\frac{\Delta t_{13}}{\Delta t_{12}}\right) \tag{4}$$

$$c(\theta) = \frac{d\cos\theta}{\Delta t_{12}} = \frac{d\Delta t_{12}}{\Delta t_{12}\sqrt{\Delta t_{12}^2 + \Delta t_{13}^2}} = \frac{d}{\sqrt{\Delta t_{12}^2 + \Delta t_{13}^2}} \tag{5}$$

From equations (4) and (5) the wave propagation direction and the wave velocity in that direction are obtained in terms of experimentally measured values Δt_{12} and Δt_{13}. If three more sensors S_4, S_5 and S_6 are mounted near another corner of the plate as shown in Figure 2 then the wave propagation direction θ_4 from the acoustic source to sensor S_4 and the wave speed in that direction $c(\theta_4)$ can be obtained in the same manner from Δt_{45} and Δt_{46} from the following equations.

$$\theta_4 = \tan^{-1}\left(\frac{\Delta t_{46}}{\Delta t_{45}}\right) \tag{6}$$

$$c(\theta_4) = \frac{d}{\sqrt{\Delta t_{45}^2 + \Delta t_{46}^2}} \tag{7}$$

From equations (1) and (4) of the S_1, S_2, S_3 sensor cluster, and from similar two equations for the S_4, S_5, S_6 sensor cluster one can write

$$\tan\theta = \frac{y_1 - y_A}{x_1 - x_A} = \frac{\Delta t_{13}}{\Delta t_{12}} \tag{8}$$

$$\tan\theta_4 = \frac{y_4 - y_A}{x_4 - x_A} = \frac{\Delta t_{46}}{\Delta t_{45}} \tag{9}$$

Equations (8) and (9) give a system of two linear equations with two unknowns x_A and y_A that can be uniquely solved. In other words, two straight lines with inclinations θ and θ_4 going through sensors S_1 and S_4 intersect at a point which is the acoustic source point as shown in Figure 2.

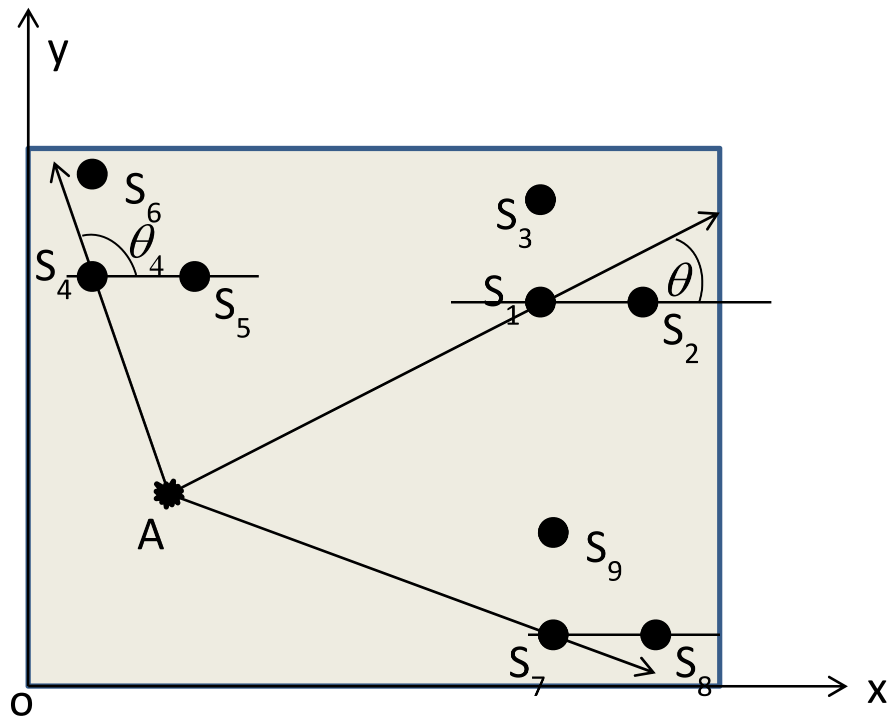

Figure 2. Three sets (or clusters) of acoustic sensors on a plate.

Determination of Δt_{ij}

It should be noted that all derived values - acoustic wave propagation direction (θ and θ_4 in Figure 2), acoustic source location (A in Figures 1 and 2) and the direction dependent wave speed $c(\theta)$ - are obtained from Δt_{ij}. Therefore, it is necessary to measure it accurately. Since the distance d between the sensors is small the time difference Δt_{ij} between two recorded signals by i-th and j-th sensors placed in close proximity is expected to be small. However, this time difference still can be accurately measured in the following manner.

Let the recorded transient signals by i-th and j-th sensors be expressed as two arrays $\mathbf{I}(t) = [I_1, I_2, I_3, \dots I_N]$ and $\mathbf{J}(t) = [J_1, J_2, J_3, \dots J_N]$. Here I_n and J_n represent the signal values at time t_n. Note that the time increment δt between two successive points in the transient signal is given by $\delta t = \frac{T}{N-1}$ where T is the total recorded time and N is the total number of points in the transient signal. These two arrays can be added and multiplied after giving a small time shift in one of the two arrays as shown below.

$$U(\Delta t) = \sum_{n=1}^{N-m} [|I_n + J_{n+m}|] \tag{10}$$

$$V(\Delta t) = \sum_{n=1}^{N-m} [I_n \times J_{n+m}] \tag{11}$$

Where

$$\Delta t = m \times \delta t \tag{12}$$

If $U(\Delta t)$ and $V(\Delta t)$ are plotted then they should reach their maximums values at $\Delta t = \Delta t_{ij}$ because then these two arrays are in phase. If two arrays in phase are added and all negative terms are made positive after addition by taking their magnitudes as shown in Eq.(10), then that value should be higher than for the same two arrays in out-of-phase positions. Same thing can be said for the two arrays when they are multiplied as shown in Eq.(11). In this manner Δt_{ij} can be measured very accurately with precision equal to δt, the time increment of the recorded transient signal.

Improving and Checking the Accuracy of Prediction

Accuracy of the prediction can be checked and improved, if needed, by using three more receiving sensors (S_7, S_8 and S_9) placed at another location of the plate as shown in Figure 2. If the third line generated by this sensor cluster goes through the intersection point of the first two lines generated by sensor clusters (S_1, S_2, S_3) and (S_4, S_5, S_6) as shown in Figure 2 then it can be concluded that the prediction is accurate and reliable. Otherwise, if the three lines form a triangle instead of coinciding at one point then there is some uncertainty associated with this prediction. In that case the intersection point of two longer lines (connecting the source and the sensor) should be considered as the impact point completely ignoring the shortest line. This is because the sensor cluster closest to the acoustic source is expected to have the maximum error since for a short distance between the sensor cluster and the acoustic source the assumption that the lines connecting the source point and the three sensors are almost parallel is violated.

For having more confidence on the prediction it is recommended that instead of 6 sensors (forming 2 clusters) 9 sensors (making 3 clusters) should be used if possible. These three clusters should be placed at three locations far from one another so that the impact location is always far from at least two clusters.

Specialization for Isotropic Plates

For an isotropic plate the acoustic source can be localized using 4 sensors instead of 6 as described here. Since for an isotropic plate the velocity of the guided wave is independent of the direction of propagation one does not need to evaluate the velocity using Eq.(7) after obtaining it from Eq.(5). However, to obtain θ_4 from Eq.(6) three sensors (S_4, S_5, S_6) are needed therefore, not using Eq.(7) does not help to reduce the number of required sensors if Eq.(6) is used. Both these equations can be avoided in the formulation for an isotropic plate in the following manner.

Instead of using the sensor arrangement shown in Figure 2, a new sensor arrangement with four sensors, as shown in Figure 3 is proposed for the isotropic plate.

Note that wave arrival times t_1 and t_4 at sensors S_1 and S_4 are related to distances D and D_4 (see Fig. 3) in the following manner

$$\frac{D_4}{c} = t_4, \qquad \frac{D}{c} = t_1 \tag{13}$$

$$\therefore \frac{D_4 - D}{c} = t_4 - t_1 = \Delta t_{14} \tag{14}$$

Or,

$$\sqrt{(x_4 - x_A)^2 + (y_4 - y_A)^2} - \sqrt{(x_1 - x_A)^2 + (y_1 - y_A)^2} = c \times \Delta t_{14} \tag{15}$$

and from Eq.(1)

$$\frac{y_1 - y_A}{x_1 - x_A} = \tan\theta \tag{16}$$

In Eqs. (15) and (16) only unknowns are x_A and y_A. These two unknowns can be evaluated from these two equations in the following manner.

Since the inclination angle θ in Fig. 3 is known, x_A and y_A values corresponding to a general point A^* on the inclined line (see Figure 3) can be obtained from Eq.(16). Substituting x_A, y_A and other coordinate values in Eq.(15) it can be checked if this equation is satisfied. In other words, it is investigated whether the difference between the lengths D_4^* and D^* of Fig.3 matches with the right hand side of Eq.(15). If they do not match then the position of A^* is changed and Eq. (15) is checked again. This process is continued until Eq.(15) is satisfied. Note that only for a single position of $A^* = A$ the following equation is satisfied, and in this manner the source is localized.

$$D_4^* - D^* = D_4 - D = c \times \Delta t_{14} \tag{17}$$

Following the above steps the acoustic source for an isotropic plate can be localized using only four sensors. A four channel oscilloscope is sufficient to record four receiving signals. However, for an anisotropic plate monitoring six sensors and a six-channel recorder are needed.

CONCLUSIONS

A new formulation for predicting the acoustic source point in a large anisotropic plate using six acoustic emission (AE) sensors is presented. The main advantage of this formulation is that it does not require the knowledge of the wave velocity in the plate. Dependence of the guided wave velocity on the wave propagation direction for anisotropic plates, and its dependence on signal frequency for both isotropic and anisotropic plates are two major obstacles for acoustic source localization in a plate. Both these obstacles are completely bypassed in this formulation. The proposed technique also does not rely on the constraint condition that the principal strain direction must coincide with the wave propagation direction, and thus can localize acoustic source in any type of anisotropic plate. As a byproduct of this formulation the angle dependent wave velocity in the plate can be obtained easily from the recorded data. For isotropic plate 4 AE sensors (instead of 6) are required for localizing the acoustic source. For experimental verification of the theoretical formulation presented here readers are referred to reference [21].

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