

# A Semi-Analytical Layerwise Wave Propagation Model for Composite Strips with Piezoelectric Actuators and Sensors and Capabilities of Damage Detection

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## ABSTRACT

A semi-analytical solution for the propagation of guided waves generated by external forces and surface tractions due to piezoelectric actuators along a semi-infinite strip is presented. The paper proposes the use of the discrete layerwise theory for modeling the displacement field through the thickness of the strip and uses a double Fourier transform for the solution of the problem in the frequency-wavenumber domain. Solutions are first presented for a healthy strip with various laminate configurations. Subsequently, solutions for a damaged strip with various sizes of a delamination crack are shown and compared with the healthy response, in order to have an estimation of the effect of damage on the wave characteristics.

## INTRODUCTION

The extensive application of composite materials in aerospace, civil engineering, transport and renewable energy structures requires new Structural Health Monitoring (SHM) techniques capable of revealing and locating damage in composite structures and ensuring their structural sustainment. The generation and monitoring of guided Lamb waves using permanently attached piezoelectric wafer and/or film actuators and sensors seems to be one of the most encouraging SHM methods for the detection, identification and localization of damage in composite structures. A critical issue which needs to be resolved is the development of modeling tools for the efficient analysis of such complex systems. To this end, this paper presents a semi-analytical solution for the wave propagation along a semi-infinite strip.

Early work in the area of guided wave propagation as a technique of damage detection was focused on the dispersive behavior of guided waves in laminated plates of finite thickness, showing that the dispersive modal propagation behavior is strongly influenced by the anisotropic properties of each lamina and the stacking sequence used. Nayfeh [7] deals with the general problem of elastic wave propagation in multi-layered anisotropic media, obtaining exact analytical solutions for the interaction of the harmonic elastic wave with the laminate, using the transfer matrix method, whereas Alleyne and Cawley [1] used a 2D Fourier transform of the time history of waves and measured the amplitudes and velocities

of the Lamb waves propagating in a plate. Pan and Datta [8] studied the guided waves and transient response of multilayered superconducting tapes, focusing mainly on the dispersion of two-dimensional guided waves. 2D Fourier transformations in both time and space were used and results concerning the dispersive characteristics of three different cases showed considerable variations. Additional Finite Element Analysis was used by Mukdadi and Datta [6] in order to study the guided waves in both infinite- and finite-width elastic plates, whereas Castaings and Hosten [2] studied the propagation of Lamb-like waves in sandwich plates made of anisotropic and viscoelastic material layers using a semi-analytical model based on the transfer matrix method. Crack detection in isotropic structures via modeling high-frequency wave propagation was attempted by Coccia et al [3, 4] in arbitrary cross-section waveguides, where certain modes were selected for the detection of surface cracks. Giurgiutiu [5] modeled surface attached piezoelectric actuators and sensors to excite and detect tuned Lamb waves for structural health monitoring.

In the rest of this paper, a semi-analytical modeling tool has been developed based on a layerwise laminate theory [10], involving an analytical wave solution in the plane of the semi-infinite strips. Surface traction excitations are implemented in the model, to model external forces or shear tractions due to piezoceramic actuators. A two-dimensional Fourier transform is used in order to transform the system equations into the frequency-wavenumber domain and the transformed solution is obtained. Various damage cases are introduced into the governing equations as the degradation of properties in a finite length of the strip. Obtained solutions for a healthy baseline model and a model with a delamination crack are presented and compared, in order to evaluate the performance of the method and to investigate the effect of delamination on the wave characteristics of the strip.

**THEORETICAL FORMULATION**

The layerwise theory for the wave propagation in semi-infinite composite strips is presented in this section. Figure 1 shows a simple baseline case of a healthy strip, as well as with a single delamination crack at the mid-span of the strip.

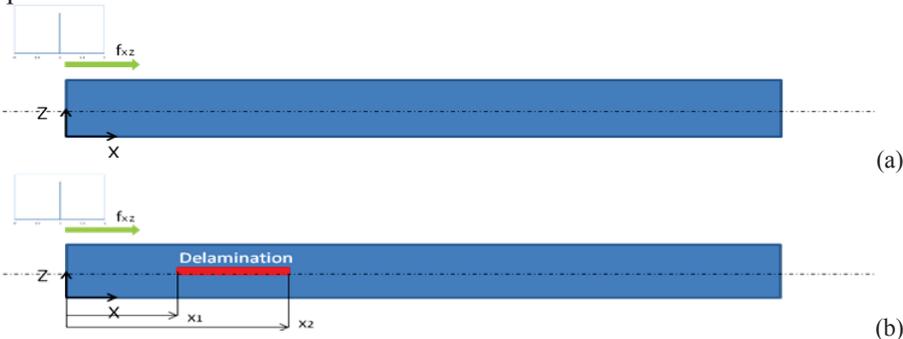


Figure 1. Composite beam (a) healthy, (b) with a delamination crack placed at the width's mid-span between  $x_1$  and  $x_2$  points along strip's length

**Kinematic Assumptions**

Kinematic hypotheses of a layerwise theory are adopted, admitting in-plane and transverse piecewise linear displacement fields through the thickness [10]. The laminate is subdivided into  $N$  discrete layers, where each discrete-layer may contain a single ply, a sub-laminate, or a sub-ply. Linear fields are assumed in

each discrete layer for the in-plane field through the laminate thickness. The displacement field in the laminate takes the form:

$$u(x, z, t) \cong \sum_{n=1}^N u^n(x, t) \Psi^n(z) \quad (1)$$

$$w(x, z, t) \cong \sum_{n=1}^N w^n(x, t) \Psi^n(z)$$

where superscripts  $n=1, \dots, N$  indicate the discrete layers through the thickness and  $\Psi^n$  are the linear interpolation functions through the laminate thickness for the  $n$ th layer, given by:

$$\Psi^n(z) = \begin{cases} \frac{z - z^{n-1}}{z^n - z^{n-1}}, & z \leq z^n \\ \frac{z^{n+1} - z}{z^{n+1} - z^n}, & z > z^n \end{cases} \quad (2)$$

### Equations of Motion

The mechanical equilibrium of the laminated strip is represented by the stress equilibrium for a plain strain problem in the  $(x, z)$  plane, given by:

$$C_{11} u_{,xx} + C_{13} w_{,zx} + C_{55} (u_{,zz} + w_{,xz}) = \rho \ddot{u} \quad (3)$$

$$C_{55} (u_{,zx} + w_{,xx}) + C_{13} u_{,xz} + C_{33} w_{,zz} = \rho \ddot{w}$$

Considering the case of cylindrical bending, where  $\varepsilon_{xy} = \varepsilon_{yz} = 0$  and taking into account the kinematic assumptions, integrating through the thickness of each discrete layer and collecting the common terms, the equations of motion are related to the resultant laminate stiffness and mass matrices as:

$$A_{11}^{mn} u_{,xx}^n + B_{13}^{mn} w_{,x}^n - D_{55}^{mn} u^n - \bar{B}_{55}^{mn} w_{,x}^n + [\Psi^m \sigma_{xz}]_{-h/2}^{h/2} = \bar{\rho}^{mn} \ddot{u}^n \quad (4)$$

$$B_{55}^{mn} u_{,x}^n + A_{55}^{mn} w_{,xx}^n - \bar{B}_{13}^{mn} u_{,x}^n - D_{33}^{mn} w^n + [\Psi^m \sigma_{zz}]_{-h/2}^{h/2} = \bar{\rho}^{mn} \ddot{w}^n$$

In the previous equations,  $m, n = 1, \dots, N$ , where the index  $n$  is summative and the generalized laminate matrices are given by:

$$A_{11}^{mn} = \int_h \Psi^m C_{11} \Psi^n dz$$

$A_{55}^{mn}, D_{55}^{mn}, B_{55}^{mn}$  and  $\bar{B}_{55}^{mn}$  are the interlaminar generalized laminate matrices defined as:

$$A_{55}^{mn} = \int_h \Psi^m C_{55} \Psi^n dz$$

$$D_{55}^{mn} = \int_h \Psi_{,z}^m C_{55} \Psi_{,z}^n dz$$

$$B_{55}^{mn} = \int_h \Psi^m C_{55} \Psi_{,z}^n dz$$

$$\bar{B}_{55}^{mn} = \int_h \Psi_{,z}^m C_{55} \Psi^n dz$$

$D_{33}^{mn}$ ,  $B_{13}^{mn}$  and  $\bar{B}_{13}^{mn}$  are the out-of-plane generalized laminate matrices given by:

$$D_{33}^{mn} = \int_h \Psi_{,z}^m C_{33} \Psi_{,z}^n dz$$

$$B_{13}^{mn} = \int_h \Psi^m C_{13} \Psi_{,z}^n dz$$

$$\bar{B}_{13}^{mn} = \int_h \Psi_{,z}^m C_{13} \Psi^n dz$$

and  $\bar{\rho}^{mn}$  is the generalized mass matrix.

The surface tractions admitted by Eqs. (4) are:

$$F_{xz}^m = [\Psi^m \sigma_{xz}]_{-h/2}^{h/2}$$

$$F_z^m = [\Psi^m \sigma_{zz}]_{-h/2}^{h/2}$$

### General Solution of the Transformed Functions

In order to solve the governing equations of motion in terms of displacement, a double Fourier transform with respect to time and space is first applied to Eq. (4), according to the analytic form:

$$U(\xi, \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(x, t) e^{i\omega t} e^{i\xi x} dx dt \quad (5)$$

This results in the following equations in the frequency-wavenumber domain in matrix form:

$$\begin{Bmatrix} U^n \\ W^n \end{Bmatrix} = \begin{bmatrix} -\xi^2 A_{11}^{mn} - D_{55}^{mn} + \bar{\rho}^{mn} \omega^2 & i\xi(\bar{B}_{55}^{mn} - B_{13}^{mn}) \\ i\xi(\bar{B}_{13}^{mn} - B_{55}^{mn}) & -\xi^2 A_{55}^{mn} - D_{33}^{mn} + \bar{\rho}^{mn} \omega^2 \end{bmatrix}^{-1} \begin{Bmatrix} F_{xz}^m \\ F_z^m \end{Bmatrix} \quad (6)$$

where  $m, n = 1, \dots, N$  and the index  $m$  is summative;  $U, W, F_{xz}$  and  $F_z$  are the coefficients of double Fourier transforms on the displacement and force components.

In order to get the frequency domain solution, the above equation (6) is solved for various values for the Fourier variables  $(\xi, \omega)$  and the displacement components are derived.

To obtain the time domain response of the structure, the inverse transform of Eq. (5) needs to be performed, given by:

$$f(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(\xi, \omega) e^{-i\omega t} e^{-i\xi x} d\xi d\omega \quad (7)$$

The integrations involved in the Eq. (7) are usually calculated numerically at discrete pairs of values for  $(\xi, \omega)$ .

### Modeling of Damage

In order to simulate laminate damage at a specific ply, the respective ply properties are degraded for a finite length along the strip. A delamination crack of length  $d = x_2 - x_1$ , as shown in Figure 1, is modeled as the degradation of mechanical properties  $G_{12}, G_{13}$  and  $E_{33}$  of the material to 10% of its initial value along the  $x$  direction, in a very thin discrete layer, representing the interphase at the location of delamination.

A general graph showing the assumed spatial property degradation is shown in Figure 2.

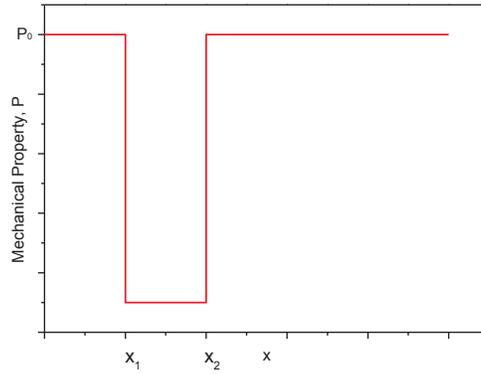


Figure 2. Assumed variation of mechanical properties along the strip

## APPLICATIONS AND DISCUSSION

In the present paper, interest is focused on the response of the strip in the frequency-wavenumber domain, comparing the healthy baseline specimen with various sizes of delamination cracks.

### Materials and Geometry

All applications presented in this section were focused on composite Glass/Epoxy strips with UD laminations, 3.7mm total thickness and semi-infinite length. The properties of the composite layers are  $E_{11} = 45GPa$ ,  $E_{33} = 13GPa$ ,  $\nu_{13} = 0.29$ ,  $G_{13} = 4.4GPa$  and  $\rho = 2000kg/m^3$ . Two different delamination crack sizes were used, placed at the mid-span of the strip. Through the thickness of the composite strip, the fields were modeled using 16 discrete uniformly spaced layers.

### Displacement Solution in Frequency Domain

Numerical results of the developed semi-analytical wave solution were extracted in the frequency-wavenumber domain, by solving Eq. (6) for various

pairs of Fourier variables  $\xi, \omega$ . During the present analysis, a concentrated Dirac horizontal force was applied at point  $x_0 = 0$  at the bottom layer of the strip and at time  $t_0 = 0$  sec in order to excite all the frequencies, having the general form:

$$f(x, t) = f^0 \delta(x - x_0) \delta(t - t_0)$$

The response of U and W displacement components of the healthy strip in the frequency-wavenumber domain for 1024 samples both for  $\xi$  and  $\omega$  variables is presented in Figure 3, at a threshold of  $thres = 1e-8m$ . At selected pairs of  $\xi$  and  $\omega$  along the curves of Fig. 3, the wave shapes through the thickness of the strip are shown in Figure 4, revealing the propagation of different Lamb wave modes, such as  $A_0, S_0, A_1, S_1$  etc.

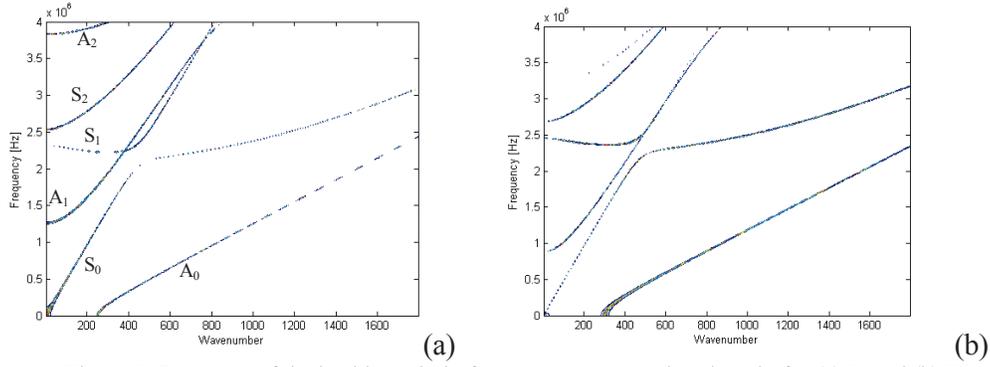


Figure 3. Response of the healthy strip in frequency-wavenumber domain for (a) U and (b) W displacement components

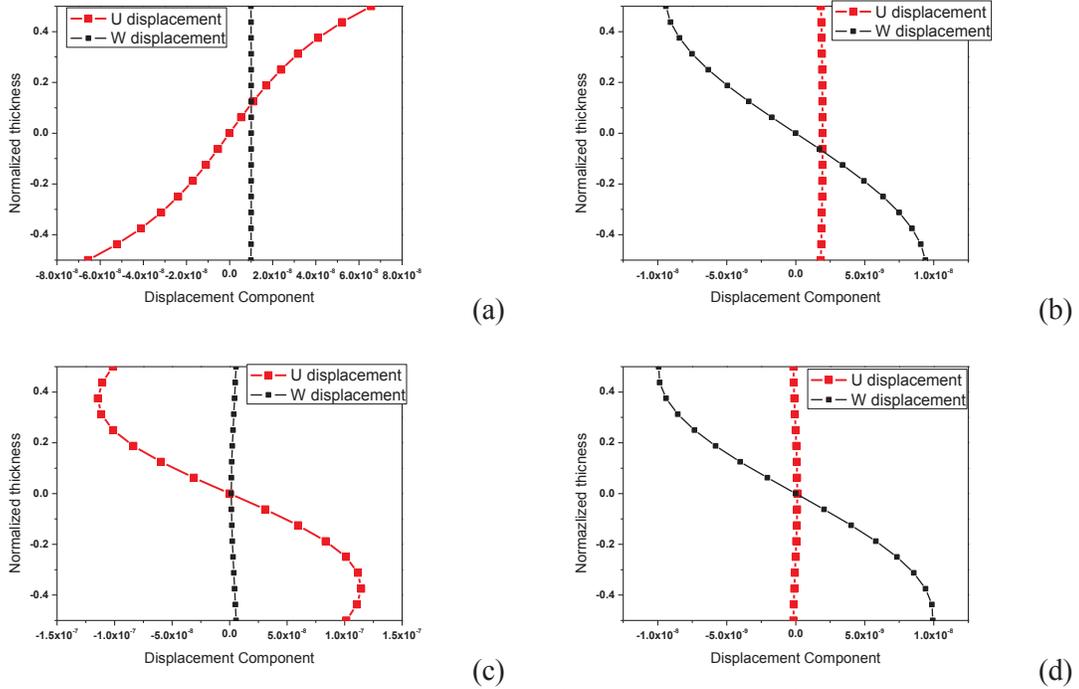


Figure 4. Wave shapes of U and W displacements through the thickness of the healthy strip showing (a) the  $A_0$  mode, (b) the  $S_0$  mode, (c) the  $A_1$  mode, (d) the  $S_1$  mode

For that reason, two different lengths of delamination cracks were modeled in the middle layers of the strip, using material properties degradation in a thin interphase layer of thickness  $h_d = 0.4265\text{mm}$ . All delaminations start at  $x_1 = 37\text{mm}$  from the left edge of the strip, i.e. ten times the total thickness of the strip and have lengths  $d=10$  and  $100\text{mm}$ .

Figure 5 presents the plots of the real part of the U displacement component at the bottom surface of the strip as calculated from Eq. (6) for various delamination sizes, for a threshold of  $thres = 1e-8m$ . From the figure it is at first obvious that, above the delamination crack, the displacement field is affected by the damage and new modes are added to the frequency output. Furthermore, the size of the crack plays an important role on the wave modes through the thickness of the strip, depicted in Figure 6, where the displacement through the thickness is plotted at selected pairs  $(\xi, \omega)$ .

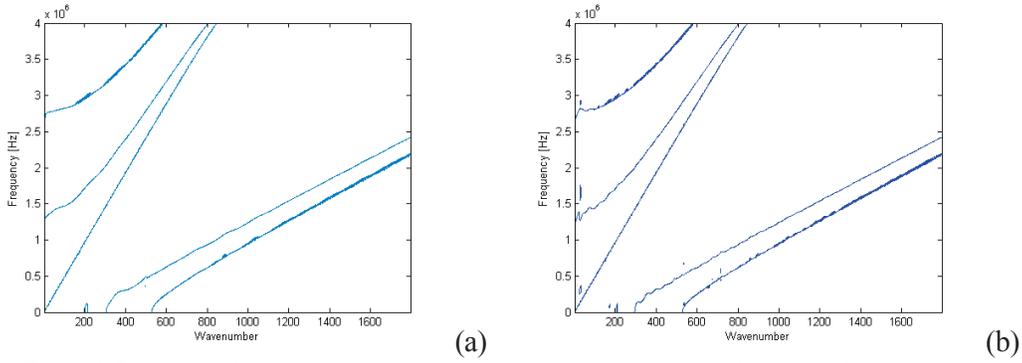


Figure 5. Results for the u displacement component at the bottom face of the strip in the frequency-wavenumber domain and for different damage sizes: (a) 10mm and (b) 100mm

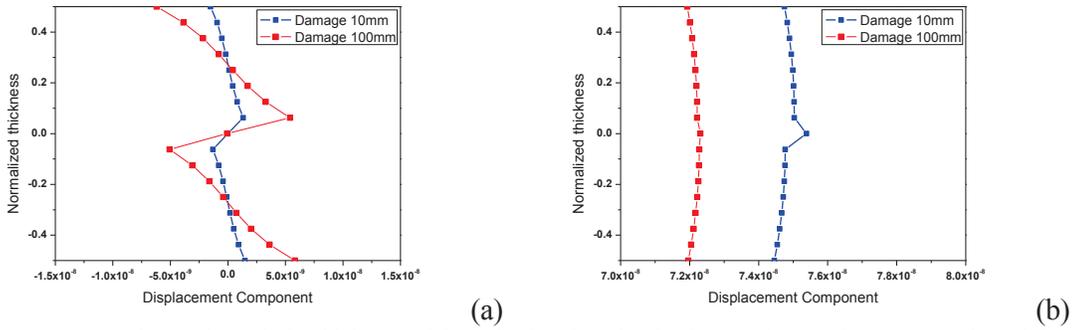


Figure 6. Wave shapes through the thickness of the strip showing for the damaged cases the (a)  $A_0$  mode and (b) the  $S_0$  mode for various frequencies,

## CONCLUSIONS

This paper presented a semi-analytical approach for the solution of the wave propagation problem along a semi-infinite strip. Kinematic assumptions using the discrete layerwise theory were used in order to approximate the displacements through the thickness, whereas a 2D Fourier transform was implemented both in time and space variables. To this end, the transformed solution of the problem in frequency-wavenumber domain was obtained. Two different configurations were examined; one healthy model and one with delamination crack. The damage was approximated assuming material degradation in a discrete layer of finite length,

simulating the size of the delamination. Results in frequency-wavenumber domain show great influence of damage existence on the wave characteristics of the model. Moreover, the proposed model seems to have promising capabilities for the qualitative estimation of damage size and location along the strip's length.

## REFERENCES

1. Alleyne, D. and Cawley, P. (1991). "A two-dimensional Fourier transform method for the measurement of propagating multimode signals", *Acoustical Society of America*, 89(3), pp. 1159-1168
2. Castaings, M. and Hosten, B. (2003). "Guided waves propagating in sandwich structures made of anisotropic, viscoelastic, composite materials", *The Journal of the Acoustical Society of America*, 113(5), pp. 2622-2634
3. Coccia, S., Bartoli, I., Lanza di Scalea, F., Fateh, M. and Carr, G. (2009). "Modeling high-frequency wave propagation in rail tracks for crack detection", *Proceedings of SPIE*, 7292
4. Coccia, S., Bartoli, I., Marzani, A., Lanza di Scalea, F., Salamone, S. and Fateh, M. (2011). "Numerical and experimental study of guided waves for detection of defects in the rail head", *NDT & E International*, 44(1), pp. 93-100
5. Giurgiutiu, V. (2005). "Tuned Lamb Wave Excitation and Detection with Piezoelectric Wafer Active Sensors for Structural Health Monitoring", *Journal of Intelligent Material Systems and Structures*, 16(4), pp. 291-305
6. Mukdadi, O. M. and Datta, S. K. (2003). "Transient ultrasonic guided waves in layered plates with rectangular cross section", *Journal of Applied Physics*, 93(11), pp. 9360-9370
7. Nayfeh, A. H. (1991). "The general problem of elastic wave propagation in multilayered anisotropic media", *Acoustical Society of America*, 89(4), pp. 1521-1531
8. Pan, E. and Datta, S. K. (1999). "Ultrasonic waves in multilayered superconducting plates", *Journal of Applied Physics*, 86(1), pp. 543-551
9. Raghavan, A. and Cesnik, C. E. S. (2007). "Review of Guided-wave Structural Health Monitoring", *The Shock and Vibration Digest*, 39(2), pp. 91-114
10. Saravanos, D.A., Heyliger, P.R. and Hopkins, D.A. (1997). "Layerwise mechanics and finite element for the dynamic analysis of piezoelectric composite plates", *International Journal of Solids and Structures*, 34(3), pp. 359-378