

# Monitoring of Flexible Support Effects Using the Responses of Multi-Span Bridges Via Wavelet Transform

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## ABSTRACT

In this paper the temporal and spectral effects of the flexible support on the response of multi-span bridges subjected to spatially-varying differential support motions are investigated by using wavelet transform. The Modified Littlewood-Paley wavelet basis is used for the analysis. In the case study of a two-span bridge, the spatially-varying earthquake motions at support bases are simulated. Two finite element-based models of the bridge, the rigid-base support model that assumes rigid soil from the foundation beds, and the flexible-base mid-support model accounting for actual soil stiffness, are analysed. In addition to shortening structural vibration period and amplifying the response amplitude, the flexible base is found to act as a time delay operator to the bridge responses.

## INTRODUCTION

The spatial-variability of ground motions have pronounced effects on the response of extended structures (Chakraborty and Basu, 2008). Most of extended structures are multiple-supported where the supports are founded on different soil conditions. The effects of variability in soil conditions on spatially-varying ground motions have been studied (Konakli and Kiureghian, 2011). However, the effects of such variability in soil conditions, especially for the soft soil, on the responses of multiple-support bridges subjected to spatially-varying excitations have rarely been reported in the literature and will be investigated in this paper with a view to monitor the support flexibility of the bridges.

The flexible base may influence both the stiffness of the multiple-support bridge and the arrival time of seismic waves. Thus, temporal and spectral properties of the bridge responses may be modified by the flexible base. A time-frequency analysis tool

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is therefore necessary and the wavelet-based analysis can meet that need (Chakraborty and Basu, 2010). In this paper, the temporal and spectral effects of the flexible base on the responses of the bridge subjected to spatially-varying excitations are investigated by using wavelet transform and the Modified Littlewood-Paley (Basu and Gupta, 1998) is chosen as the wavelet basis.

In the case study of a two-span bridge, the spatially-varying earthquake motions at support bases are simulated. Two finite element-based models of the bridge, the rigid-base support model that assumes rigid soil from the foundation beds, and the flexible-base mid-support model accounting for actual soil stiffness, are analysed. In addition to shortening structural vibration period and amplifying the response amplitude, the flexible base is found to act as a time delay operator to the bridge responses.

## MODELING OF A MULTI-SPAN BRIDGE WITH FLEXIBLE SUPPORT UNDER SPATIALLY-VARYING SUPPORT MOTIONS

Consider a bridge having  $N$  degrees of freedom (DOFs) and  $n$  supports subjected to excitation time histories  $u_g = \langle u_{g1} \ u_{g2} \ \dots \ u_{gN} \rangle$ , in which  $u_{gr}$  is different from those at the other supports. The bridge is modelled in a finite-element (FE) framework leading to a discrete dynamical system model. Assuming that the effect of entire velocity-damping coupling is negligible in comparison to that of the inertia, the motion equations of the bridge is given by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -[\mathbf{M}\mathbf{E} + \mathbf{M}_g]\ddot{\mathbf{u}}_g \quad (1)$$

where,  $\mathbf{u}(t)$  represents the displacement vector relative to the support motions and  $\mathbf{M}$ ,  $\mathbf{C}$ , and  $\mathbf{K}$  are the system  $N \times N$  mass, damping and stiffness mass matrices, respectively. In Eq. (1),  $\mathbf{E} = -\mathbf{K}^{-1}\mathbf{K}_g$  is the  $N \times n$  influence coefficient matrix, the  $N \times n$  matrices  $\mathbf{M}_g$  and  $\mathbf{K}_g$  accounts for the coupling of the inertia and stiffness between structural DOFs and ground motion DOFs.

To facilitate modal analysis, the assumption of lumped mass matrix has been widely adopted. Consequently, the coupling of the inertia between structural and ground motion DOFs,  $\mathbf{M}\mathbf{E}\ddot{\mathbf{u}}_g$  in Eq. (1) disappears. In this paper, the consistent mass matrix is used. Thus, the inertia coupling term  $\mathbf{M}\mathbf{E}\ddot{\mathbf{u}}_g$  is included in Eq. (1). Moreover, the flexible bases are accounted for by modifying the stiffness matrix and the boundary conditions of the rigid-base structure. The stiffness matrix of a flexible support structure is no longer modal-decoupled and direct integration method is used to solve the motion equations (1).

In the flexible-base support, actual soil stiffness and damping coefficients are frequency-dependent. For simplification, a constant vertical stiffness (Wolf and Deeks, 2004) of the soil base can be assumed as in Eq. (2) where  $L$  and  $B$  are the dimensions of the rectangular base and  $G_s$  is the shear modulus of the soil.

$$K_{sz} = \frac{4G_s r_0}{1 - \nu_s}, \quad r_0 = \sqrt{\frac{L \times B}{\pi}} \quad (2)$$

## SIMULATION OF SPATIALLY-VARYING NON-STATIONARY MOTIONS

The simulation of spatially-varying non-stationary subsurface motions (Dinh et al, 2012) are briefly presented. Earthquake motions at  $n$  subsurface sites,  $\ddot{u}_{g1}(t)$ ,  $\ddot{u}_{g2}(t)$ , ...,  $\ddot{u}_{gn}(t)$ , can be considered as components of the one-dimensional multivariate (1D-mV) non-stationary zero-mean stochastic vector processes having diagonal and off-diagonal elements of the cross-spectra density matrix as

$$S_{jj}^0(\omega, t) = |A_j(\omega, t)|^2 S_j(\omega) \quad (3)$$

$$S_{jk}^0(\omega, t) = A_j(\omega, t)A_k(\omega, t)\sqrt{S_j(\omega)S_k(\omega)}\gamma_{jk}(\omega), \quad j, k = 1, 2, \dots, n; j \neq k \quad (4)$$

where  $A_j(\omega, t)$  and  $S_j(\omega) = |H_j^{subs}(\omega)|^2$  are the amplitude-and frequency-modulation and the stationary power spectral density function of  $\ddot{u}_{gj}(t)$ , respectively. The term  $H_j^{subs}(\omega)$  is the Fourier amplitude spectrum of earthquake motions at subsurface sites that can be represented by using the stochastic seismic spectrum that consists of the scaling factor, the two-corner frequency source spectrum, geometrical spreading function, path-dependent attenuation, diminution factor accounts for the path-independent attenuation of high-frequency waveforms, and the amplification factor approximated by the source-to-site impedance ratio.

The term  $\gamma_{jk}(\omega)$  in Eq. (4) is the complex coherency function between  $\ddot{u}_{gj}(t)$  and  $\ddot{u}_{gk}(t)$ ,  $\gamma_{jk}(\omega) = |\gamma(\xi_{jk}, \omega)| \exp[i\theta(\xi_{jk}, \omega)]$ , where  $\xi_{jk}$  is the separation distance between sites  $j$  and  $k$ . The complex part of  $\gamma_{jk}(\omega)$  is the coherency phase that is incorporated by the wave passage effect and arrival-time perturbations in this paper as  $\theta(\xi_{jk}, \omega) = -\omega(\xi_{jk}/V_s + \Delta t_{r,jk})$  with  $\Delta t_{r,jk}$  being a zero mean normally distributed random variable. The real part  $|\gamma(\xi_{jk}, \omega)|$ ,  $0 \leq |\gamma(\xi_{jk}, \omega)| \leq 1$  called the lagged coherency characterizes the variation in space. The functional form of Harichandran and Vanmarcke (1986) lagged coherency (H-V1986) model is adopted with subsurface site parameters estimated by Dinh et al. (2012).

$$|\gamma(\xi, f)| = A \exp\left[\frac{B(\xi)}{\alpha\sigma(f)}\right] + (1 - A) \exp\left[\frac{B(\xi)}{\sigma(f)}\right], \quad (5)$$

$$B(\xi) = -\frac{2\xi}{k}(1 - A + \alpha A), \quad \sigma(f) = \left[1 + \left(\frac{f}{f_0}\right)^\beta\right]^{-\frac{1}{2}} \quad (6)$$

The 1D-mV algorithm (Deodatis 1996) is then introduced to simulate the motions. To provide temporal non-stationary, common forms of intensity modulation can be used together with the total duration at a site (Dinh et al., 2012) that is contributed by both source and path durations.  $A_j(\omega, t)$  can be the parametric time- and frequency-modulation (Chakraborty and Basu, 2008).

## CONTINUOUS WAVELET TRANSFORM

Consider a “mother” wavelet function,  $\psi(t)$ , having finite energy. A family of baby wavelets can be constructed by scaling and translating  $\psi(t)$  using the dilation (or scale) parameter ‘ $a$ ’ and the translation parameter ‘ $b$ ’ as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \quad (7)$$

The parameter  $b$  localises the basis function at  $t = b$  and its neighbourhood, where  $a$  controls the frequency content of the basis function by stretching or compressing it (with the number of cycles remaining unchanged).

The continuous wavelet transform of the finite energy process  $u(t)$  with respect to the basis  $\psi(t)$  is obtained by convolving the signal  $u(t)$  with a set of its baby wavelets

$$W_{\psi}u(a,b) = \frac{1}{\sqrt{|a|}} \int u(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (8)$$

where (\*) denotes the complex conjugate. Eq. (8) gives the localized frequency information of  $u(t)$  around  $t = b$ . The wavelet transform coefficient,  $W_{\psi}u(a,b)$ , represents how well the signal  $u(t)$  and the scaled and translated mother wavelet match. More significantly,  $W_{\psi}u(a,b)$  represents the contribution to  $u(t)$  in the neighborhood of  $t = b$  and in the frequency band corresponding to the value of  $a$ . The scale and translation parameters can be numerically assumed as  $a_j = \sigma^j$  and  $b_i = (i-1)\Delta b$  where  $\sigma$  and  $\Delta b$  are constant parameters.

In this paper, the Modified Littlewood-Paley (MLP) (Basu and Gupta, 1998) is chosen as the wavelet basis because it provides high accuracy in spectral analysis and advantages in numerical computation. The MLP wavelet basis pairs is given by

$$\psi(t) = \frac{1}{\pi\sqrt{2F_1(\sigma-1)}} \cdot \frac{\sin(2\pi F_1\sigma) - \sin(2\pi F_1 t)}{t} \quad (9)$$

$$|\hat{\psi}(\omega)| = \frac{1}{\sqrt{4\pi F_1(\sigma-1)}}, \quad F_1 \leq \left|\frac{\omega}{2\pi}\right| \leq \sigma F_1$$

$$= 0 \quad \text{otherwise} \quad (10)$$

where  $F_1$  is the initial cut-off frequency of the mother wavelet. It is noted by Basu and Gupta (1998) that  $\sigma = 2^{1/n}$ ,  $n \geq 4$  is found reasonable based on investigations on several ground motions recorded. However, as small value of  $\sigma$  leads to increased computational effort,  $\sigma = 2^{1/4}$  has been chosen.

## CASE STUDY

A two-span simply-supported bridge crossing over a river as shown in Figure 1a is considered. The two abutments and the mid-support having rectangular bases are supported by bored piles reaching a hard soil layer at the depth  $-50$  m. The bridge has span length  $L = 88$  m, inertia moment and area of the cross section  $I_c = 5.33 \text{ m}^4$  and  $A_c = 4.0 \text{ m}^2$ , elastic modulus  $E = 2.0 \times 10^{11} \text{ N/m}^2$ , Poisson ratio  $\nu = 0.29$ , mass density  $\rho = 7860 \text{ kg/m}^3$ , and modal damping ratio  $\zeta_1 = \zeta_2 = 0.02$ .

As the two abutments rest on large groups of piles in order to avoid differential settlement with the approach slabs, the two end-supports of the bridge can be assumed to rest on rigid bases. The mid-support resting on a smaller group of piles can be modeled as a *rigid-base support* (RBS, Figure 1b) or a *flexible-base mid-support* (FBMS, Figure 1c). The first five natural frequencies of the RBS model are 1.182, 1.846, 4.727, 5.984 and 10.647 Hz. Those of the FBMS model with  $L = B = 2$  m are 1.182, 1.274, 3.128, 4.727 and 7.551 Hz. The inclusion of flexible base at the mid-support reduces significantly the natural frequencies of all vibration modes except the first, which is dominant and asymmetric.

The time-histories of the spatially-varying non-stationary accelerations (SVNA) at there subsurface sites beneath the support are simulated by using Eqs. (3-6). The parameters estimated at the separation distance 88 m for the H-V1986 lagged coherency model are  $A = 0.4567$ ,  $\alpha = 0.00798$ ,  $\beta = 6.848$ ,  $f_0 = 2.608$  Hz, and  $k = 1.799 \times 10^5 \text{ m}$  (Dinh et al. 2012). A single realization of acceleration at site 1 and site 2 is given in Figures 2(a, b), respectively.

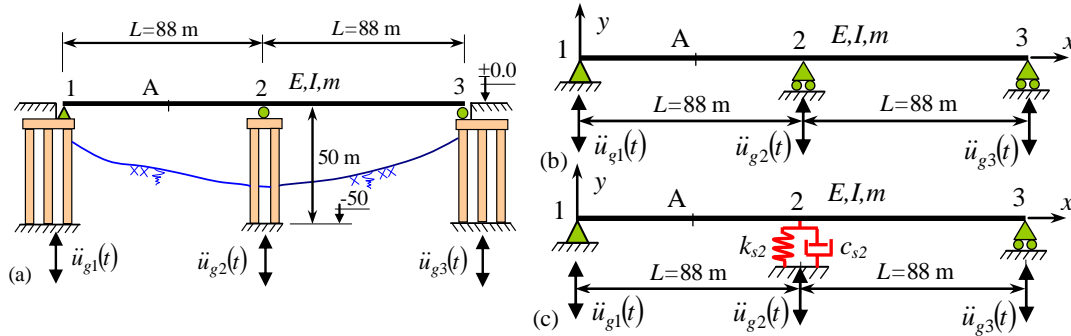


Figure 1. Two-span bridge: (a) Geometry; (b) RBS model; (c) FBMS model.

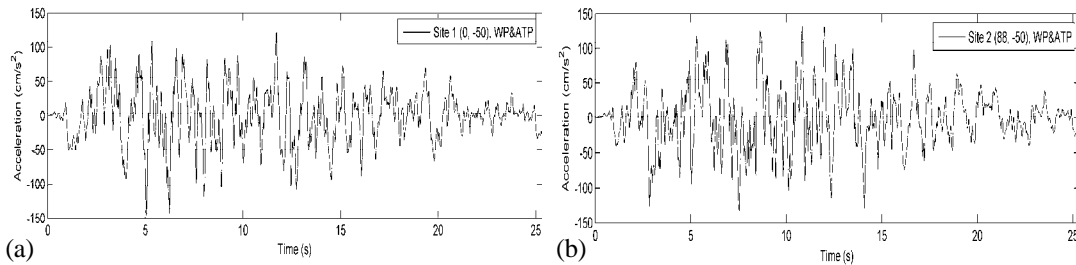


Figure 2. Samples of acceleration excitations at foundation bases: (a) site 1 and (b) site 2.

Figure 3 shows the computed time histories of the left mid-span vertical displacement of the RBS model and FBMS model by SVNA excitations. The vibration periods of the FBMS model are considerably larger that of the RBS model.

Contrary to the response of the RBS model, the amplitude of the FBMS model is much amplified in the second half of the duration. When the soil damping is excluded, the FBMS model amplitude peaks are larger than that of the RBS model.

To monitor both temporal and spectral effects of the flexible base on the bridge responses, the computed time histories of the responses are transformed into wavelet domain by using Eq. (8) and the MLP wavelets. Having observed from the Fourier amplitude spectra of excitations presented by Dinh et al. (2012), the most of the energy input lies in the frequency range 0 to 15 Hz. As shown in Table 2, the five fundamental natural frequencies of both RBS model and FBMS model are also in that range. To cover such a frequency range, the scale parameters selected are  $a_j = \sigma^j$ ,  $\sigma = 2^{1/4}$  and  $j = -24, +3$  and  $F_1 = 1.5$  Hz.

Figures 4(a-b) shows the squared wavelet coefficients of the left mid-span vertical displacement of the RBS and the FBMS models, respectively. In the RBS model, all peaks occur in the frequency 6 to 14 Hz, the highest peak corresponds to a frequency of about 11 Hz and appears early in the time duration 0 to 12 s. Whereas in the FBMS model, all peaks occur in the narrower frequency range 6 to 10 Hz, the highest peak corresponds to a lower frequency of about 6.5 Hz and appears later in the time duration 10 to 25 s. These wavelet analyses results can be used as an indicator for monitoring the health of the bridge with respect to the flexible support effects.

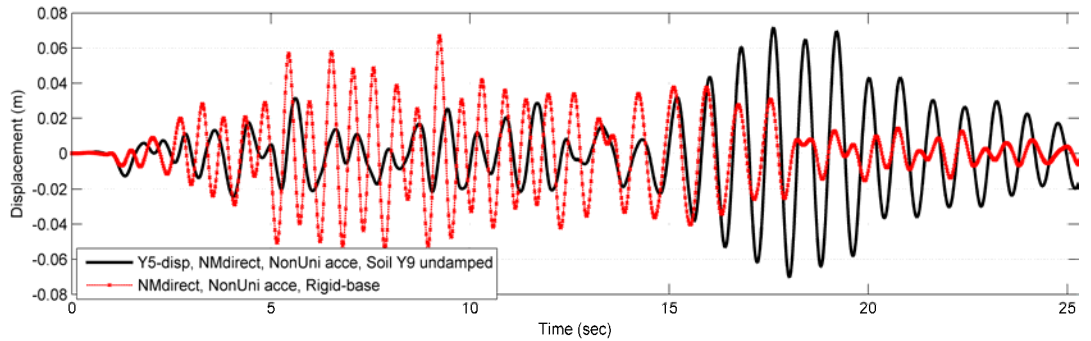


Figure 3. Vertical displacement at the left mid-span of of FBMS model with undamped soil and base  $2 \times 2$  m (continuous heavy line) and RBS model (red dashed x), SVNA excitations.

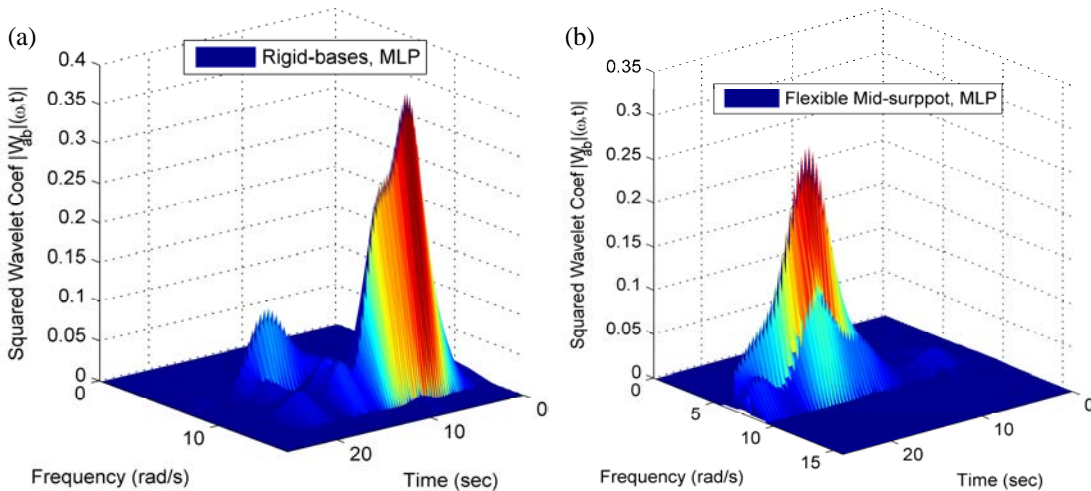


Figure 4. Squared wavelet coefficients of vertical displacement at the left mid-span, SVNA excitations; (a) RBS model, and (b) FBMS model with undamped soil and base  $2 \times 2$  m.

## CONCLUSIONS

The temporal and spectral effects of the flexible base on the responses of multi-span bridges subjected to spatially-varying differential support motions have been investigated by using wavelet transform with the Modified Littlewood-Paley wavelet basis. In the case study of a two-span bridge, the spatially-varying earthquake motions at three support bases are simulated. Two finite element-based models of the bridge, the rigid-base support model that assumes rigid soil from the foundation beds, and the flexible-base mid-support model accounting for actual soil stiffness, are analysed. The wavelet analysis of the vibration responses of the bridges can be used to monitor the health of a bridge with respect to the flexibility of the foundations.

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