Zero-Pad Effects on Conditional Simulation and Application of Spatially-Varying Earthquake Motions

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ABSTRACT

Realistic simulated ground motion are required for testing heath monitoring algorithms in connection with seismic excitations. To this end spatially-varying ground motions are important for long span structures such as bridges and pipelines. In conditional simulation of spatially-varying earthquake motions, a single earthquake motion time history at a site is known and a parametric coherency model is given. Spatially-varying earthquake motions at other sites are then simulated. To facilitate the use of the Fast Fourier Transform (FFT), dummy zero points must be added to the front and/or the end of the data. Investigating effects of zero-pad is important for simulating ground motions to be used for testing health monitoring techniques. However, neither comparison of zero-padded filtered data with non-zero pad filtered data nor explanation for effects of zero pads have been reported. The zero-pad effects on FFT of the input motion and also on the simulated motions are examined in this paper. When the input motion is a recorded acceleration time history, the Fourier amplitude spectra of the zero-padded input and simulated accelerations have lower peaks and higher resolution compared to the non zero-padded. The two phenomena are due to the contribution of the added data points and the conservation of the power energy when the data length is increased, respectively. The effects of zero pad on structural responses due to the simulated earthquake motions are also investigated by using a single degree of freedom model, where significant influence on the time history of structural responses at certain structural frequencies are observed.

INTRODUCTION

To facilitate the use of FFT in simulating earthquake motions for structural health monitoring (SHM) applications, dummy zero points must be added to the data, in the front and/or the end, as shown in Figure 1. At first, we consider a simple

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example of an 1-D array A_1 [1 1 1 1], its FFT is [4 0 0 0]. If four more zeros are padded at the end, A_{10} [1 1 1 1 0 0 0 0], the FFT becomes

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[4.0 1.0-2.4142i 0 1.0-0.4142i 0 1.0+0.4142i 0 1.0+2.4142i]
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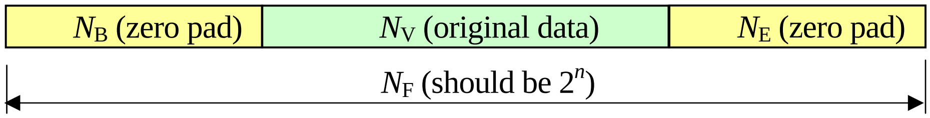

Figure 1. Schematic chart of zero-padded data.

The FFT of zero-added data A_{10} is different from that of the original A_1. Because zero-padding is necessary for the FFT algorithm, examining effects of zero-pad is important. In this conditional simulator, the input data of acceleration, through its Fourier transform in frequency domain, is filtered by a coherency function and/or by a phase-variation function. The zero-pad effects on input data and also on simulated acceleration should be examined. Newland (1993) suggested a correction factor for correlation of two series due to zero-pads. Boore (2005, 2009) and Zerva (2009) analyzed these effects when processing raw earthquake records. However, neither comparison of zero-padded filtered data with non-zero pad filtered data nor explanation for effects of zero pads have been reported in the literature.

SIMULATION OF SPATIALLY-VARYING EARTHQUAKE MOTIONS

Having assumed that earthquake-induced motions are space-homogenous, the n^{th} Fourier coefficient $F_k(\omega)$ of motion $a_k(t)$ at a station k can be evaluated by

$$F_k(n) = F_j(n)\left|\gamma_{jk}(n)\right| \exp\left[-i\omega_n\left(t_{jk}^{wp} + \Delta t_{jk,n}^{r}\right)\right] \tag{1}$$

where, $F_j(n)$, $\left|\gamma_{jk}(n)\right|$, t_{jk}^{wp} and $\Delta t_{jk,n}^{r}$ are, respectively, the n^{th} Fourier coefficient of motion at the source station j, the n^{th} lagged coherency value, the time lag due to wave-passage effects and the n^{th} value of random arrival-time perturbation series.

The motion at a discrete time $t_r = r\Delta t$, $r = 0, \ldots, (N-1)$ is

$$a_k(t_r) \equiv a_{k,r} = \sum_{n=0}^{N-1} F_k(n) e^{i(2\pi nr/N)} \tag{2}$$

The time-history of a signal $\{a_k(t)\}$ and its frequency-history Fourier transform $\{F_a(\omega)\}$ represent the same signal, thus the energy in the time-history and Fourier transform must be the same (Donelly and Rust, 2005). If the real input data is considered, this energy equality can be expressed by using Parseval's relation as in Eq. (3) that suggests an overall check for the accuracy of Fourier transform as well as a methodology to explain for the deviations in amplitudes of Fourier transform.

$$\sum_{n=0}^{N-1} a_k^2 = \frac{1}{N}\left(\left|F_0\right|^2 + 2.\sum_{m=1}^{(N/2)-1}\left|F_m\right|^2 + \left|F_{N/2}\right|^2\right) \tag{3}$$

CASE STUDIES

Zero-pad Effects on Conditional Simulation of Earthquake Motions

In this case study, the El Centro 1940 North-South ground acceleration shown in Figures 2(a-c) is taken as the input data at station *k*. The acceleration at station *j* is simulated by using Eq. (2). Energies of input time-history and its Fourier transform, and generated time-histories and their Fourier transform are computed by using Eq. (3) and presented in Table 1, which show that zero-pads in either front or the end of the original data do not influence on the energy of the simulated accelerations.

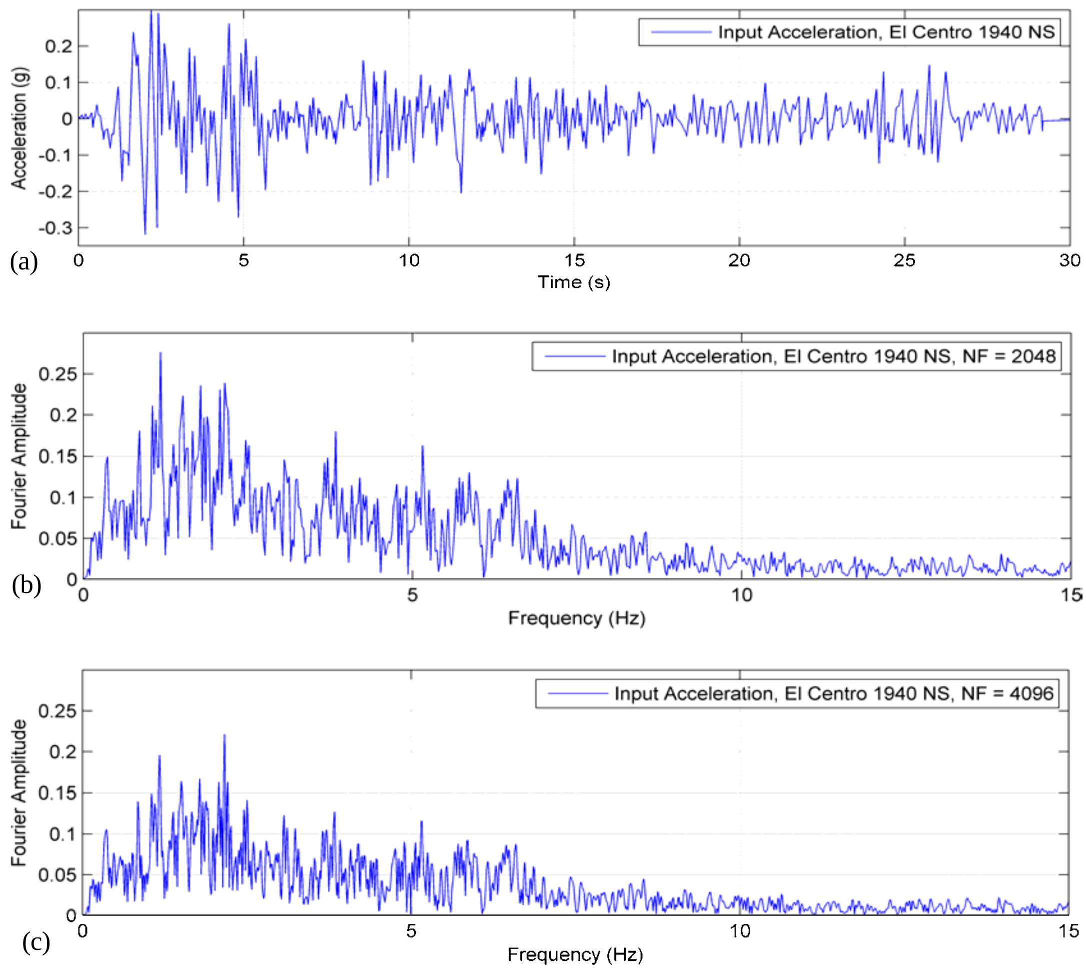

Figure 2. Input acceleration (El Centro 1940 NS, N_V = 1559): (a) Time history, (b) Fourier amplitude spectrum with NF = 2048 and (b) Fourier amplitude spectrum with NF = 4096.

Table 1. Energy of various zero-pad patterns with El Centro 1940 N-S input data.

Energy unit = g^2	Original input (N_V = 1559, N_F = 2048, N_B = 0, N_E = 489)		More zero-pad (N_V = 1559, N_F = 4096, N_B = 0, N_E = 2537)	
	Time-history	Fourier Trans.	Time-history	Fourier Transform
Input acceleration	5.8457	5.8457	5.8457	5.8457
Simulated, delay excluded	3.6727	3.6727	3.6727	3.6727
Simulated, uniform delay	3.6727	3.6727	3.6727	3.6727
Simulated, all time delays	3.6727	3.6727	3.6727	3.6727

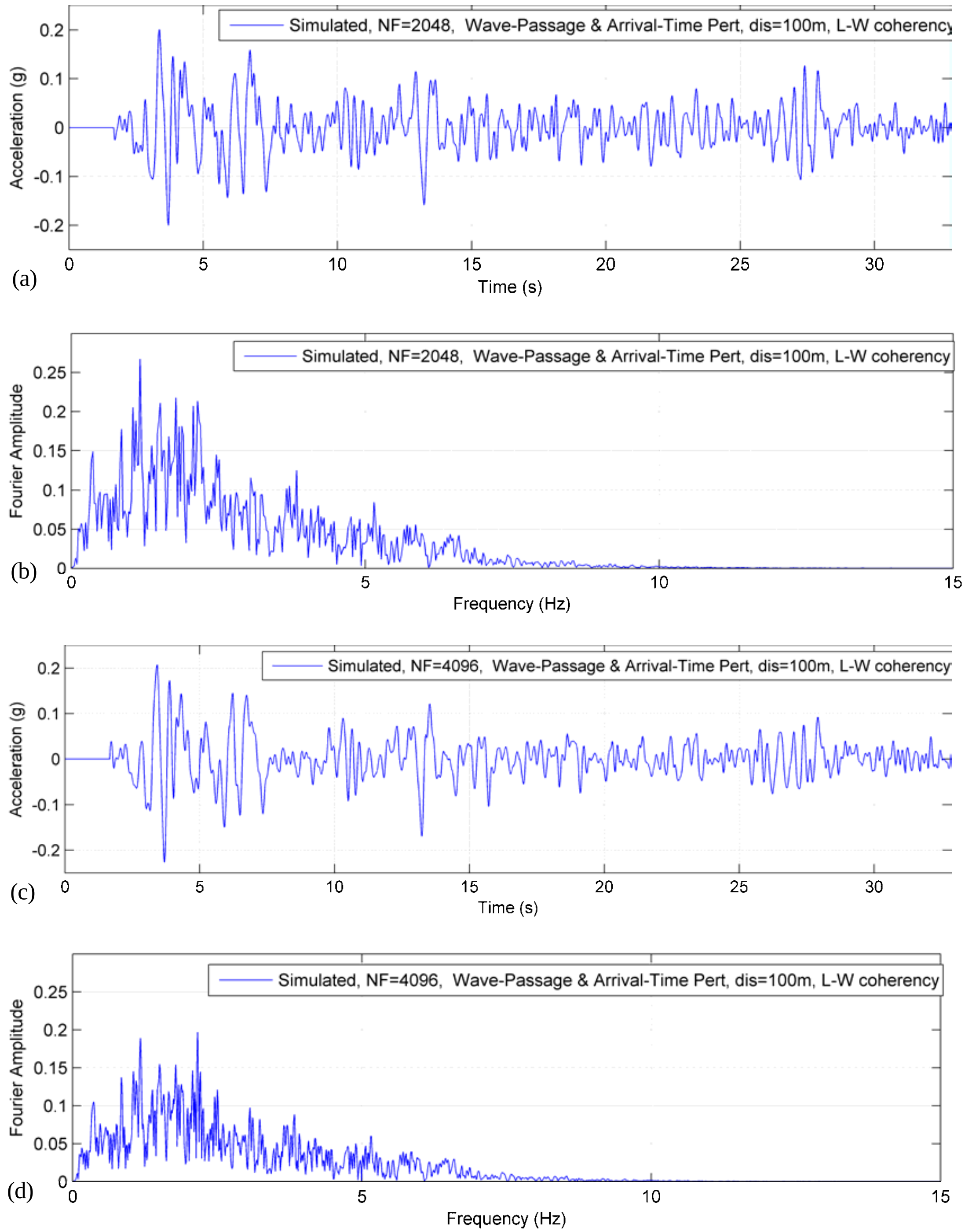

Figure 3. Simulated accelerations at 100 m with end-zeros padded: (a) Time history for N_F = 2048, (b) Fourier amplitude of (a), (c) Time history for N_F = 4096 and (d) Fourier amplitude of (c).

The accelerations at station *j* simulated from El Centro 1940 N-S component record with two zero-pad patterns (N_F = 2048 and N_F = 4096) are shown in Figures 3(a-d). No effect of zero-padding on the simulated acceleration time-histories is observed from Figures 3(a, c). When more zeros (N_F = 4096) are padded, the Fourier amplitude spectrum of input in Figure 2c is denser (i.e. resolution is higher) than the spectrum in Figure 2b (N_F = 2048). The Fourier amplitude spectrum of simulated acceleration in Figure 3d also is denser than the spectrum in Figure 3b. This is due to the contribution of added number of data points. Besides that, when more zeros are padded, the Fourier amplitude peaks of input and simulated accelerations are considerably smaller. This reduction in amplitude peaks is due to conserving the power when the length of the data series is increased.

Zero-pad Effects on Responses of a SDOF under Simulated Earthquake Motions

The simulated spatially-varying earthquake motions at *n* sites are used as the input for the seismic analysis of multiple degrees of freedom (MDOF) structures having *n* supports. The structure motion equations can be transformed into a set of uncoupled modal equations. Thus, in order to investigate the effects of zero pad on structural responses due to the simulated earthquake motions, the single degree of freedom (SDOF) motion equation in Eq. (4) can be used with $\omega_s = 2\pi f_s$ and ξ_s denoting the natural frequency and damping ratio, respectively and $\ddot{u}_{gi}(t)$ the single simulated earthquake motion of interest.

$$\ddot{y}(t) + 2\xi_s \omega_s \dot{y}(t) + \omega_s^2 y(t) = -1 \cdot \ddot{u}_{gi}(t) \tag{4}$$

The SDOF motion equation in Eq. (4) is considered with the damping ratio ξ_s = 0.02 and two values of natural frequencies f_s = 1.2 Hz and f_s = 2.2 Hz. The two frequencies correspond to the peaks in the Fourier spectra of the simulated accelerations, Figures 3b and 3d. The computed time histories of the displacement response $y(t)$ and the corresponding Fourier amplitude spectra for two patterns of zero padding N_F = 2048 and N_F = 4096, are plotted in Figures 4, 5, 6 and 7.

For the lower frequency f_s = 1.2 Hz as shown in Figures 4 and 5, the computed time history of displacement response and the corresponding Fourier amplitude spectrum for two patterns of zero padding are similar. Hence, the effects of zero padding on structural responses are not observed at this frequency. For the higher frequency f_s = 2.2 Hz as shown in Figures 6 and 7, the Fourier amplitude spectrum for two patterns of zero padding are similar. However, the computed time histories of displacement responses are significantly different. Thus, zero padding influences significantly the time history of structural responses.

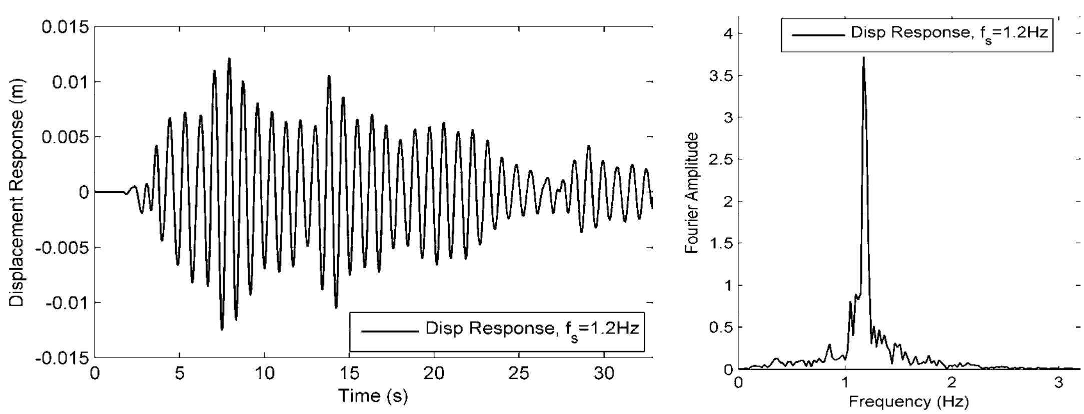

Figure 4. Time history and Fourier amplitude of displacement response, f_s = 1.2 Hz, N_F = 2048.

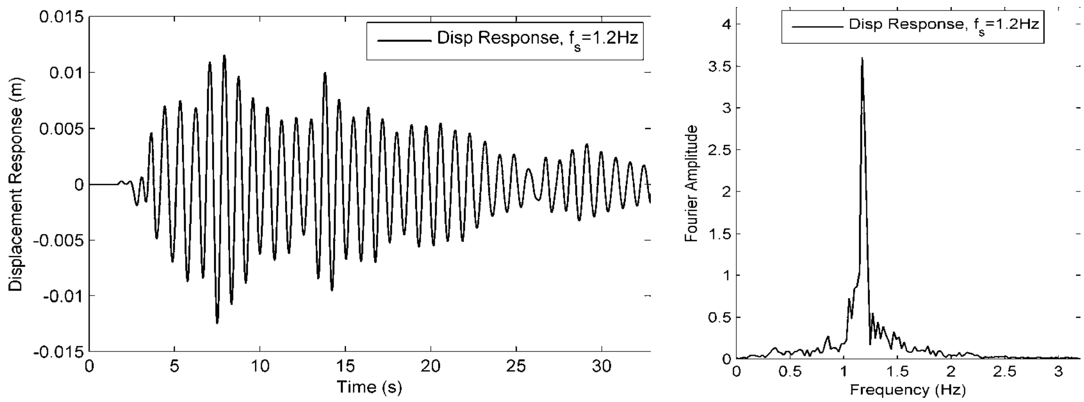

Figure 5. Time history and Fourier amplitude of displacement response, f_s = 1.2 Hz, N_F = 4096.

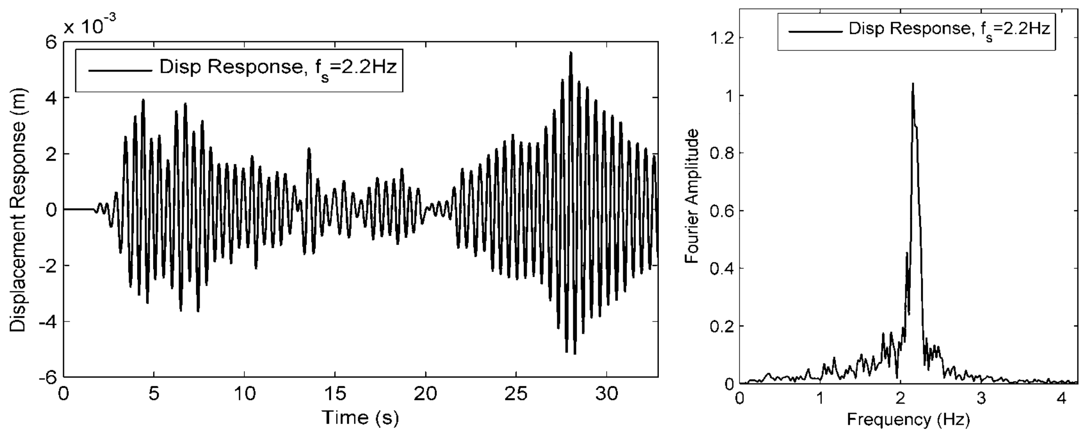

Figure 6. Time history and Fourier amplitude of displacement response, f_s = 2.2 Hz, N_F = 2048.

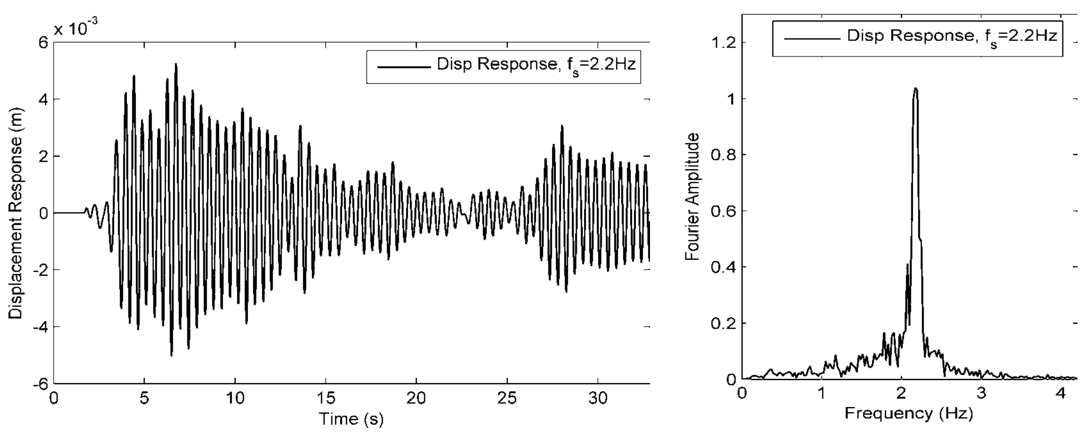

Figure 7. Time history and Fourier amplitude of displacement response, f_s = 2.2 Hz, N_F = 4096.

CONCLUSIONS

The effects of dummy zero data points added to the front and/or the ends of the data to facilitate the use of the Fast Fourier Transform (FFT) in simulating earthquake motion for structural health monitoring (SHM) application have been investigated. The zero-pad effects on the input motion and also on the simulated motions are examined. When the input motion is a recorded acceleration time history, the Fourier amplitude spectra of the zero-padded input and simulated accelerations have lower peaks and higher resolution compared to the non zero-padded. The two phenomena are due to the contribution of the added data points and the conservation of the power energy when the data length is increased, respectively. The effects of zero pad on structural responses due to the simulated earthquake motions are also investigated by using a single degree of freedom model, where significant influence on the time history of structural responses at certain structural frequency are observed.

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