Modal Piezoelectric Transducers with Shaped Electrodes for Improved Passive Shunt Vibration Control of Smart Piezo-Elastic Beams

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ABSTRACT

Modal control and spatial filtering technologies for vibration and/or structural acoustics radiation mitigation may be implemented through the use of distributed modal piezoelectric transducers with properly shaped electrodes which, in order to increase the robustness and stability of the controlled structural system, turn undesirable mode's contributions unobservable and uncontrollable over the bandwidth of interest. In addition, distributed modal piezoelectric transducers may also yield a higher generalized modal electromechanical coupling coefficient which is an important design parameter to take into account for a proper and efficient passive shunt damping design. The improvements in passive shunt damping performance when using modal piezoelectric transducers with shaped electrodes are investigated in this article for a two-layered resonant-shunted piezo-elastic smart beam structure. The damping performance of the shunted smart beam with shaped electrodes is investigated and assessed in terms of the generalized modal electromechanical coupling coefficients and frequency responses obtained when considering uniform and modally shaped electrodes, and the underlying improved performance and advantages are assessed and discussed.

INTRODUCTION

The high applicability and interest on the use of piezoelectric materials for the control of vibration and/or structural acoustics radiation of flexible structures are nowadays well recognized. Piezoelectric transducers are more commonly used under active vibration control frameworks. However, piezoelectric transducers may also be connected to passive electrical circuitry with electrical impedance defined by the resistive, capacitive and inductive elements. In this process energy is converted from mechanical to electrical form in the piezoelectric transducer and by coupling it with a passive electrical network, with properly designed electrical impedance, the converted

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energy is conveniently tackled so that stiffness and damping modifications of the structural system occur. Thus, it comes as no surprise that the electromechanical energy conversion efficiency is of paramount importance in this process for a well-succeeded and damping-efficient approach. This latter strategy was coined as *piezoelectric shunt damping* [6] and due to its passive nature it strongly differs in simplicity of use and implementation when compared with active control strategies, somewhat avoiding bulky amplifying, signal conditioning and control logic systems [8].

Concepts of modal control and shaped transducers have gained increased popularity. The investigation on design techniques and optimized polarization profiles for distributed shaped sensors in beam [7], plate [2] and shell [3] structures, and the experimental application and investigation of these concepts, for the vibration control of beam [1] and plate [11] structures, have been presented recently. Applications of these technologies may include not only vibration and/or structural acoustics control, but also structural health monitoring [5] and energy harvesting applications, among others. In the context of shunt damping, the vibration suppression of a hard disk driver actuator arm using a topology-optimized piezoelectric transducer was investigated in [10] and the passive damping and exact annihilation of vibrations of beams using shaped piezoelectric layers and tuned inductive networks was investigated in [9].

From the analysis of the open literature, it stands out that sufficient attention has not been given neither to the actuation efficiency and utility of modal actuators nor to the underlying physics of the electromechanical coupling efficiency of piezoelectric transducers with shaped electrodes. Properly designed spatially shaped sensors may yield a sensing behaviour with improved modal electromechanical coupling coefficients, which may be used to increase the shunted damping performance. This last aspect, while addressed to some extent in a few of works [9, 10], was not properly explored in the context of shunt damping applications using modal piezoelectric transducers with shaped electrodes. It represents the main motivation and novelty of this article and is investigated here for beam structures with spatially shaped uniform and single-mode electrode designs of resonant shunted piezoelectric transducers, the main aim being to demonstrate the improved damping efficiency resulting from the use of modally shaped electrode designs for vibration control purposes. An electromechanical one-dimensional equivalent single-layer Euler-Bernoulli analytical model of two-layered smart piezo-elastic beams with arbitrary spatially shaped electrodes is used (see [15] for further details), the damping performance of a shunted smart beam with shaped electrodes is investigated and assessed in terms of the generalized electromechanical coupling coefficient, and frequency responses obtained when considering uniform and modally shaped electrodes and the underlying improved performance and advantages are assessed and discussed.

SPATIALLY SHAPED SMART PIEZO-ELASTIC BEAM

Let us consider a generic beam structure of total length l with a distributed piezoelectric transducer/layer mounted on one side of the beam (Figure 1). The piezoelectric layer is assumed to be perfectly bonded onto the beam and the electrodes are spatially shaped so that they have an arbitrary width, $2b_{\mathrm{e}}(x)$, which is non-uniform along the length coordinate of the beam, x. The subscripts $(\cdot)_{\mathrm{b}}$, $(\cdot)_{\mathrm{p}}$ and $(\cdot)_{\mathrm{e}}$ are used here to denote quantities associated with material and geometric properties of the elastic and piezoelectric layers and with the electrodes, respectively. The cross-section of the elastic (bare beam) and piezoelectric layers is defined by their thicknesses, $2h_b$ and $2h_p$, and their width, $2b_b$ and $2b_p$. This two design possibilities, i.e., uniform (constant) and non-uniform (varying) width of the electrodes are the subject under study in this article and are designated in what follows as *uniform* or *modal* (if a modal-based shape is adopted) transducers.

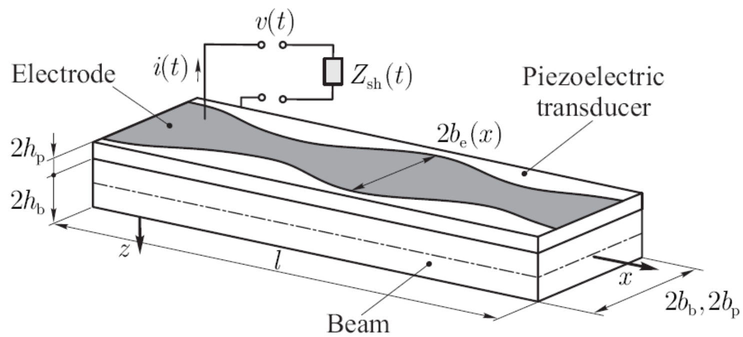

Figure 1. Generic two-layered smart piezo-elastic beam structure with a distributed shunted piezoelectric transducer with arbitrary spatially shaped electrodes.

The stiffness and mass effects of the elastic (or host beam structure) and piezoelectric layer are both kept in the formulation but it is assumed that rotary inertia effects may be neglected. A piezoelectric material of the crystal class $mm2$, polarized in the transverse direction, and one-dimensional reduced actuating and sensing constitutive behaviours are considered. The equipotentiality of the prescribed/measured voltage, $v(t)$, across the electrodes for open-circuit electric boundary condition [14] is considered using x-independent analytical definitions of the voltage and charge variables.

MATHEMATICAL MODEL AND RESONANT SHUNT DAMPING

Due to lack of space the derivation of the mathematical modelling and governing equations is not detailed here. The interest reader is referred to [15] for detailed derivations and further details on these important aspects, which will serve as the departure point for the results and analysis presented in the following section. Nonetheless, the theories and assumptions considered are briefly described here. The partial differential equations governing the transverse vibration of a generic two-layered smart piezo-elastic beam structure (Figure 1), i.e., the boundary-value problem and spatially weighted distributed piezoelectric actuation and sensing equations, were obtained in [15] following the well-known Euler-Bernoulli theory for thin beams and through the use of the extended Hamilton principle, applied to electromechanical continua. Since the piezoelectric electrode shape considered in this work is not constant and its width is allowed to vary along the length of the beam (Figure 1), another aspect which is important here is the electrode design for spatial weighting and filtering. Thus, modal and uniform piezoelectric transducer and spatial filtering designs, considering whether the smart piezo-elastic beam with a distributed piezoelectric transducer with arbitrarily spatially shaped electrodes or with a uniform (constant width) distributed electrode, are considered in [15], where the corresponding electric charge and current sensing equations for each case are derived. The piezoelectric shunt damping physics for the shunted modal and uniform piezoelectric transducer cases and the corresponding modal and frequency response models, along with considerations and a discussion on resonant shunt damping and how optimal electrical circuit parameters may be determined, are also presented in [15].

APPLICATION AND RESULTS

Let us consider a cantilever two-layered piezo-elastic smart beam with the geometry and materials depicted in Figure 1 and defined in Table 1. The analytical model results are generated considering a truncated undamped modal model with the first four modes being used. For the numerical results a three-layered (two layers for the beam and one for the piezoelectric transducer) through-the-thickness discretization and 100 (equally spaced) elements along the beam length are considered to define the one-dimensional finite element model used; it follows a first-order shear deformation partial discrete-layer (or layerwise) theory [13]. The structural, sensing and actuating dynamics of the models previously described were verified and validated in [13, 15].

Table 1. Geometry and material of the considered smart piezo-elastic elastic beam.

	Beam		Transducer	
Thickness /mm	$2h_{\mathrm{b}}$	2	$2h_{\mathrm{p}}$	0.5
Width /mm	$2b_{\mathrm{b}}$	30	$2b_{\mathrm{p}}$	30
Length /mm	l	300	–	–
Density /$\mathrm{kg\,m^{-3}}$	ρ_{b}	2710	ρ_{p}	7800
Young modulus /GPa	Y_{b}	70	Y_{p}	73.26
Piezoelectric constant /$\mathrm{C\,m^{-2}}$	–	–	e^{*}_{31}	−20.476
Dielectric constant /$\mathrm{nFm^{-1}}$	–	–	ϵ^{*S}_{33}	9.789

Generalized Electromechanical Coupling Coefficient

In what follows the comparison of uniform and modal electrode designs will be performed by investigating the generalized electromechanical coupling coefficients so obtained. Let us consider the two-layered smart beam previously defined considering two design possibilities for the electrodes, i.e., uniform and modal electrode configurations. We may consider that the electrodes are short-circuited, and in that case it re-

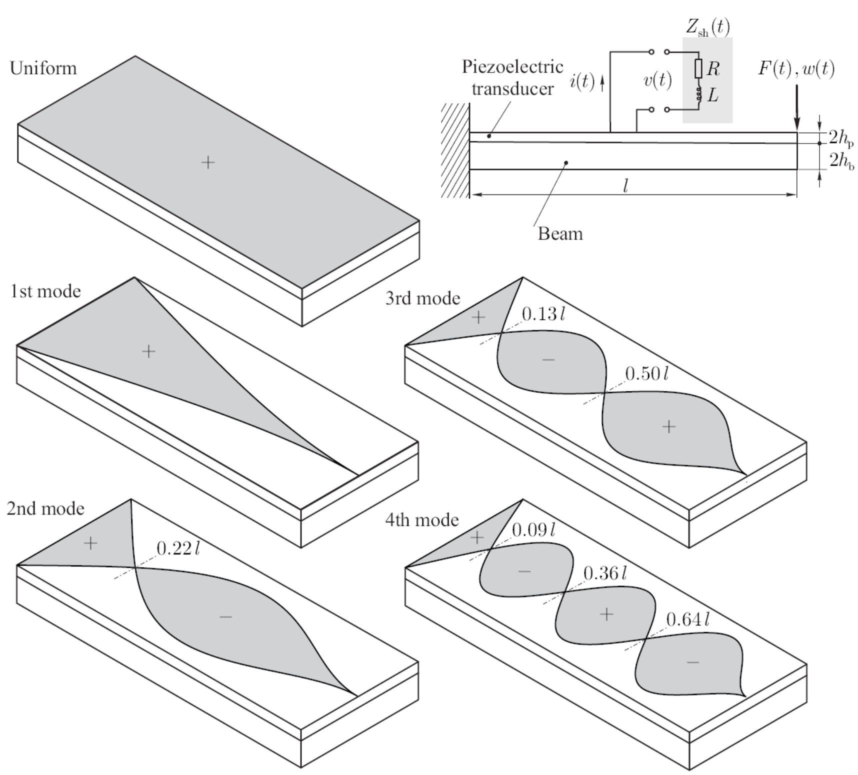

Figure 2. Case study schematic; uniform and modal electrode configurations.

Table 2. Analytical and numerical generalized electromechanical coupling coefficients (%), i.e. χ_r, of the cantilever smart piezo-elastic beam considering uniform and modal electrode shapes.

	Uniform		Modal							
			1st mode		2nd mode		3rd mode		4th mode	
Mode	1D-A	1D-N	1D-A	1D-N	1D-A	1D-N	1D-A	1D-N	1D-A	1D-N
1	37.3	33.9	38.0	36.6	0	0	0	0	0	0
2	20.7	19.1	0	0	36.3	35.1	0	0	0	0
3	12.1	11.3	0	0	0	0	36.0	34.8	0	0
4	8.66	8.11	0	0	0	0	0	0	35.9	34.6

1D-A: one-dimensional analytical; 1D-N: one-dimensional numerical.

sults in a zero electric field across the electrodes, but it does allow electric charge to flow from the positive terminal to the negative one. On opposition, we may consider that the electrical terminals are open such that no charge can flow between the electrical terminals. In general, the mechanical stiffness increases when the boundary condition changes from closed- (or short-) to open-circuit conditions. This gives rise to a stiffness modification of the structural system resulting in two distinct values for the natural frequencies of a certain mode. This latter aspect can be used to alternatively define the generalized electromechanical coupling coefficient, χ_r, in terms of the relative frequency variation between the two electrical boundary conditions [6, 12], and is the approach followed here to determine χ_r from the results obtained from the one-dimensional finite element analysis. The analytical and numerically predicted generalized electromechanical coupling coefficients, considering uniform and modal electrode shapes, are presented in Table 2.

On a first analysis we look at the estimation of the coefficients, a slight difference being obtained between the analytical and numerical values which becomes more significant for the first mode where, as previously discussed, the open-circuit electric boundary condition is tackled in a discretized manner which certainly yields some difference in results. However, the difference is only relevant for the first mode, and mostly in the uniform case, and the same trend is observed. In addition, and more importantly, it turns out that for the modal case higher values of the coefficient are obtained, the difference being more significant as the correspondent modal curvatures involve more and more nodal points for which the reverse polarity possibility of the modal designs reveals more advantageous. It should be noted also that a small increase in the coefficient is obtained for the first mode and that for the second to fourth modes the coefficient increases roughly two, three and four times, respectively, that of the uniform case. These results certainly allow us to envision that better shunt damping performances would be obtained with the modal design as compared with the uniform one which, depending upon the type of boundary condition and mode number, may suffer from a phenomenon known as charge cancelation effect [4, 12]. This last issue, as regards the shunt damping performance, is analyzed in the following section.

Comparison of Resonant Shunt Damping with Uniform and Modal Electrodes

In this section a comparison of the resonant shunt damping performance between the two electrode designs is considered. For the purpose of this comparison only the results obtained with the analytical model detailed in [15] and considering the optimal values of the single-mode shunt resistance and inductance, R_r^{opt} and L_r^{opt}, are presented here. Thus, considering these optimal values the derived modal and uniform frequency response models may be used to determine the shunt damping performance of the two electrode designs [15]. The driving-point receptance FRFs at the free end of the beam, zoomed to the specific mode being damped and tuned by the single-mode resonant shunt damping strategy, are presented in Figure 3. The optimal parameters used to tune the shunt impedance to damp each mode are presented in Table 3.

Table 3. Optimal electrical parameters used to define the passive shunt damping electrical network: optimal values of the resistor (Ω) and inductor (H) of the passive electrical network and capacitance of the piezoelectric transducer (nF).

	Uniform			Modal (Figure 3)			Modal (Figure 4)		
Mode	R_r^{opt}	L_r^{opt}	C_{p}	R_r^{opt}	L_r^{opt}	C_{p}	R_r^{opt}	L_r	C_{p}
1	21306	328	176.2	55260	833	68.98	55260	328	68.98
2	2058	9.12	176.2	7771	19.6	75.60	7771	9.12	75.60
3	443	1.20	176.2	2709	2.46	76.93	2709	1.20	73.93
4	163	0.31	176.2	1365	0.64	77.62	1365	0.31	77.62

The results in Figure 3 evidence that excellent damping performances are obtained for the two electrode design cases. As expected through the analysis of the generalized electromechanical coupling coefficients in Table 2, the difference between the two cases is notorious in all modes but the first one, where both designs conduct to similar values of the coefficient (the value of $\chi_1 = 37.3\%$ for the uniform case increases to $\chi_1 = 38\%$ for the modal case). Worthy to analyze are the optimal values of inductance necessary to tune the shunt damping electrical network to the specific mode to be damped. As seen in Table 3, the capacitance, C_{p} , is higher for the uniform case and does not vary from mode to mode. Conversely, the capacitance for the modal case is mode-dependent and is always lower than that for uniform electrodes. As a consequence, the determined (capacitance dependent) optimal resistor and inductance values are higher than the ones needed for the uniform electrodes cases, which represents a shortcoming of the modal approach. In addition, it is well known that higher values of

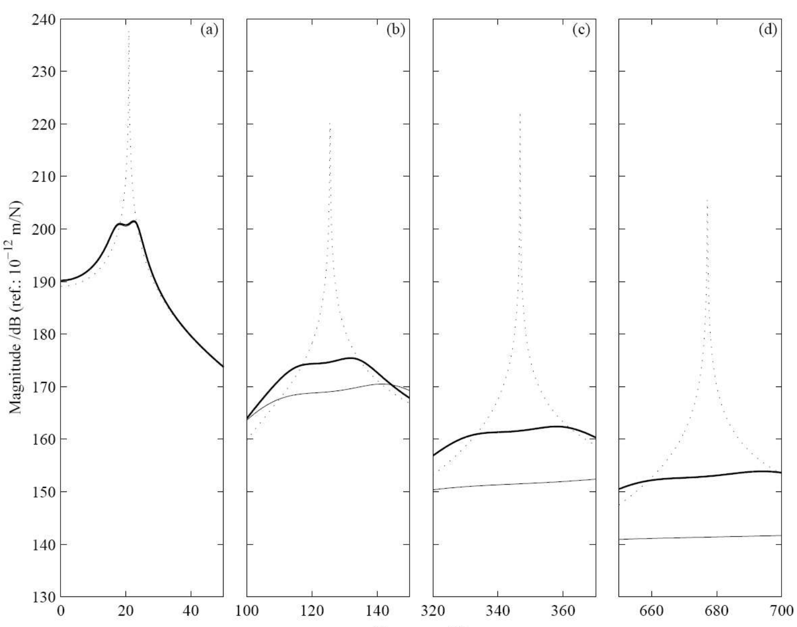

Figure 3. Frequency response function (driving-point receptance) of the clamped smart piezo-elastic beam with optimal shunt damping parameters for the uniform and modal electrode cases when the shunt damping strategy is tuned to mode 1 to 4 (left to right): open-circuit/uniform (dotted line), shunted/uniform (thick line) and shunted/modal (thin line) cases (optimal values of inductance used).

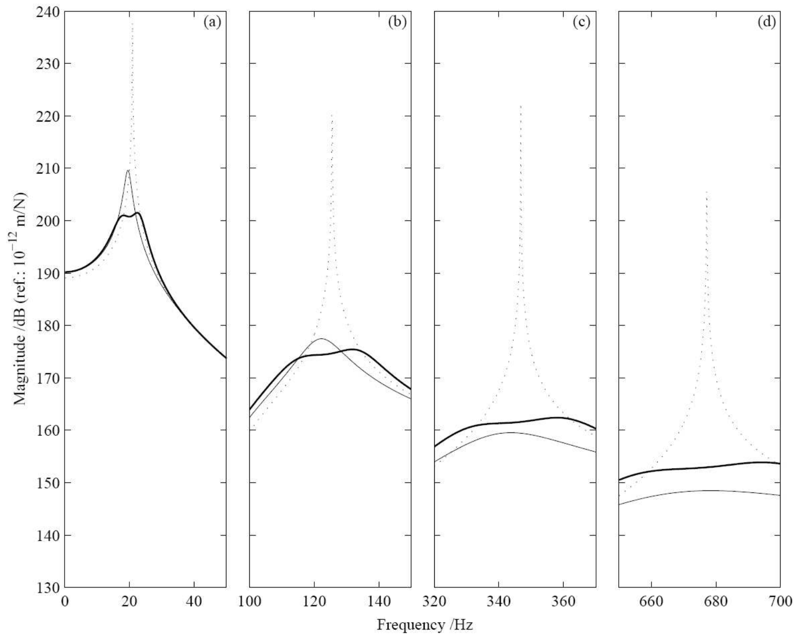

Figure 4. Frequency response function (driving-point receptance) of the clamped smart piezo-elastic beam with optimal shunt damping parameters for the uniform and modal electrode cases when the shunt damping strategy is tuned to mode 1 to 4 (left to right): open-circuit/uniform (dotted line), shunted/uniform (thick line) and shunted/modal (thin line) cases (modal inductances equal to the ones for the uniform case).

these parameters are necessary as the order mode decreases. While higher resistor values do not pose feasibility and practical implementation problems when tuning is performed at low frequencies, high inductance values up to thousands of Henry may be necessary to damp low order modes so that the weight and volume of such inductors may make unfeasible the use of resonant shunt damping in practice.

Since the value of the inductance necessary is a key aspect for the implementation of resonant shunt damping, an additional case where "non-optimal" inductance values are defined for the modal case, as presented in Table 3, is considered next. These values are in fact the same values used for the uniform case, which allows us to establish a relationship between the uniform and modal cases as far as the required inductance is concerned. For that purpose, the driving-point receptance frequency response functions (FRFs) presented in Figure 3, but with the "non-optimal" inductance values previously discussed, are now presented in Figure 4. The results show that there is a notorious degradation of damping performance for the first mode when considering a "non-optimal" value of inductance, where the value of the generalized modal electromechanical coupling coefficient does not differ significantly. However, the damping performance degradation is less significant for the second mode and increased damping is obtained for the third and fourth modes. Thus, for higher order modes, where the increase in the coupling coefficient is more significant, the modal approach is still more efficient than the uniform one, and the miss-tuned values of the resonant shunt damping approach still yield good damping performances, the same miss-tuned behaviour probably not being obtained for weakly coupled design scenarios.

CONCLUSION

This article addresses the use of passive shunt damping strategies employing piezoelectric transducers with shaped uniform and modal electrode designs for the vibration control of smart piezo-elastic beams. It is shown that, as compared with uniform designs, where charge cancelation effects may take place, for higher-order modes (higher frequencies) modal electrode designs may yield a significant increase in the generalized electromechanical coupling coefficient and that, as expected, an improved shunt damping behavior for these modes is obtained. In addition, due to the increased generalized electromechanical coupling coefficient, it is shown that the circuit is less sensible to the electrical frequency mistuning and that non-optimal inductance values may be used, somewhat compensating the capacitance decrease/inductance augmentation shortcoming of the use of shaped electrode concepts. The observed improved generalized electromechanical coupling coefficient due to the modal electrode designs yields a more efficient mechanical to electrical energy conversion, rendering interesting and appellative features not only in vibration and/or structural acoustics control contexts but also in, e.g., power harvesting applications.

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