

# Physics-Based Output-Only Model Identification of Reinforced Concrete Structures from Static Response

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## ABSTRACT

In this paper, a model-updating approach based on "output-only" measurements without knowledge of acting forces is used to identify the bending stiffness distribution of undamaged and damaged reinforced concrete (RC) beams. For this purpose static (or quasi-static) responses are utilized. Numerical evaluations which are performed in a two-step process are presented: In the first step, quasi-static structural responses acting as substitute for measurements taken on a real bridge structure are computed by performing physical nonlinear FE analyses on multiple scenarios comprising undamaged and damaged RC beams and different loads. As a second step the identification of these scenarios is performed by calibrating models which now employ linear elastic behavior. To describe regions with degraded bending stiffness, stiffness reduction functions are established. Goal of these simulations is to evaluate, whether the approach is capable to distinguish between damages to the beams and merely cracked regions.

### INTRODUCTION

Traditionally, the health state of bridges is periodically assessed by engineers who apply methods of nondestructive testing. However, depending on the knowledge and expertise of the involved engineers, the results of these assessments are mostly subjective. To support traditional bridge inspections in an objective manner, structural health monitoring (SHM) techniques are proposed. An approach to SHM is the calibration of physics-based models of the structure, which is also referred to as "model-updating" or "model adaptation".

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Model-updating means to develop an initial numerical model of the structure (baseline model, usually by employing the finite element (FE) method) and subsequently modify the properties (updating parameters) of this model iteratively, until the difference between the predicted responses of the model and the corresponding responses observed on the real structure is minimized. By interpreting the adapted models' properties, structural damages can be identified.

Most model-updating approaches are based on detecting damages by measurement and analysis of dynamic structural responses, for example natural frequencies, mode shapes or mode shape curvatures [7]. However, the use of dynamic responses has several drawbacks: (1) Global dynamic responses such as the lower eigenfrequencies are usually not affected by local damages, and (2) dynamic responses are strongly influenced by environmental effects, for example temperature variations [3]. An alternative to the use of dynamic attributes are static or quasi-static responses. One advantage of using static data is that static responses are more sensitive to damages than dynamic reactions [5].

Both dynamic and static responses can be obtained either from forced or ambient tests. When forced tests are performed, the bridge usually is closed for traffic. In forced dynamic tests, rotating mass exciters, shakers or impact hammers are used to excite the structure. In forced static tests (load tests), the bridge is loaded by vehicles with known weights. The advantage of forced tests is that the responses (output) as well as the load parameters (input) can be measured, so the model-updating can be performed from "input-output" measurements (see Figure 1). When ambient tests are performed, the structural responses are measured while the bridge is in service and subjected to traffic and wind loads. In ambient tests, the loads cannot be measured, hence the model parameters have to be identified from "output-only" measurements (see Figure 1).



Figure 1. Parameter identification from input-output (left) and output-only measurements (right) with a known model structure [2].

In recent research conducted by the authors, a long-term model-updating approach suitable for detection of damages in reinforced concrete (RC) structures has been developed [6]. The approach is based on calibration of parameterized FE models by comparing predicted and observed quasi-static responses. To ensure that the method can be applied in a long-term monitoring strategy, the model parameters are calibrated from output-only measurements. Therefore, not only uncertain system properties, but also acting traffic loads are identified. Because of the complexity of the search space, a global optimization technique based on evolutionary algorithms is employed. To take into account the nonlinear load bearing behavior of RC members due to cracking of the concrete and yielding of the reinforcement, the fitness of the solution candidates obtained during the optimization process is computed by performing physical nonlinear FE analyses. In nonlinear FE analyses of concrete structures, realistic (nonlinear) material models are applied. That means, that the properties of the

material and, hence, the stiffness of the finite elements depend on the elements' strains and vice versa. For example, when the tensile stresses of a concrete element exceed the tensile strength of the material, the stiffness of the respective elements will be consequently reduced due to cracking of the concrete.

The proposed approach as previously described includes the following advantages: (1) Since physical nonlinear FE analyses are performed, the structural behavior of RC structures due to stiffness degradation and the resulting force distribution between concrete and reinforcement can be simulated in detail. (2) Since the reinforcement is included in the FE model, the effect of reinforcement damages (e.g. loss in cross sectional area) can be evaluated as well. On the other hand, the main disadvantage of the proposed approach is the high computational cost of the physical nonlinear FE analyses. For example, physical nonlinear FE analysis of the beam model investigated later in this paper takes about 5 minutes; in contrast, linear elastic analysis of the same beam model takes only a few seconds. Hence, from the computational point of view, linear elastic FE analyses are more effective than physical nonlinear calculations.

Aim of the presented paper is to evaluate, how far linear elastic models can be utilized to distinguish between stiffness degradation resulting from concrete cracking and stiffness degradation due to damages, which had been introduced into the reinforcement of a RC structure. Commonly, when models with linear elastic behavior are adapted, the updating parameters are directly related to the stiffness of the finite elements. By assigning an updating parameter to each finite element, the stiffness of any element can be varied independently. However, this would lead to a large number of parameters and, therefore, ill-conditioning of the problem. To avoid this, the number of updating parameters should be kept at minimum through careful selection [4]. By establishing appropriate stiffness reduction functions, the stiffness distribution of a structure can be described by a small number of updating parameters. Stiffness reduction functions (also related to as "damage functions") had been introduced in [1] and [8]. In these studies the authors use damage functions with curved or piecewise linear shape to describe the stiffness distribution of RC beams. However, in their studies cracked portions of the beams are identified only. "Real" damages to the beams, for example corrosion of the reinforcement, are not investigated.

In this paper, a model-updating approach based on piecewise linear stiffness reduction functions is presented and evaluated by means of numerical simulations. The evaluations are performed in a two-step process. In the first step, quasi-static responses (namely deflections, strains, inclinations and reaction forces) are computed by performing physical nonlinear FE analyses on reinforced concrete beams with different damages under different load cases. These responses serve as substitute for measurements taken on a real bridge structure. From the results of the FE analyses, the actual bending stiffness distributions of the beams are computed and investigated regarding their identifiability. In the second step, a baseline FE model is constructed and parameterized in terms of (1) stiffness reduction functions, which describe cracked or damaged portions of the models, and (2) variables which describe the acting load. Subsequently, the stiffness reduction functions and the load attributes are identified by updating the baseline model considering the previously computed responses. After identification of the stiffness and load parameters for individual scenarios, the results are compared in order to draw conclusions on the identifiability of cracked and damaged portions of the beams.

#### **STEP 1 – GENERATION OF MEASUREMENT DATA**

In the presented studies, undamaged and damaged small-scale two-span reinforced concrete beams representing bridge structures are investigated numerically. The undamaged beam, which is referred to as "B1", is presented in Figure 2. Based on the undamaged beam, two damaged variations "B2" and "B3" are developed. "Damages" are introduced by removing 25% of the cross sectional area of the lower reinforcement in the center of the left span (in case of B2) and the upper reinforcement at the middle support (in case of B3), respectively. All beams B1, B2 and B3 are subjected to a single force F representing a single vehicle on a bridge. The force is applied in the center of the left span. In preliminary investigations it was found that the ultimate value for F (the force, that causes the beam to fail) was about 65 kN for the beam B1, 49 kN for B2 and 62 kN for B3. Based on the ultimate load level for the undamaged beam B1, two service load levels are assumed by about 60 % and about 40% of the ultimate load. Hence, three load cases, namely "L1" with F = 25 kN, "L2" with F = 40 kN and "L3" with F = 49 kN, are obtained. The combination of all beams with all load cases leads to a total of nine scenarios, namely "B1-L1", "B1-L2", "B1-L3", ..., "B3-L3". For example, scenario B1-L1 is assigned to the undamaged beam which is loaded with F=25 kN.



Figure 2. Baseline model of the investigated two-span RC beams.

By performing physical nonlinear FE analyses of all nine scenarios, different sets of response "measurements" are obtained, namely strains, deflections, rotations and reaction forces. Strains, deflections and rotations are measured by "virtual" strain gauges, displacement transducers and inclinometers, which are evenly distributed along the beams with distances of 200 mm. The reaction forces of the beams are measured by virtual load measuring bearings. For the analyses, the FE software package TNO DIANA is employed. The FE model consists of 96 2-node beam elements (element type L7BEN). The reinforcement is included as smeared reinforcement.

Due to cracking of the concrete, the bending stiffness of the cross section of a beam decreases at any location where the tensile stresses of the concrete exceed the tensile strength of the material. This behavior can be considered by performing physical nonlinear FE analysis. Resulting from the load, at the location x of the beam the bending moment M(x) occurs, which leads to curvature  $\kappa(x)$  of the cross section. The curvature  $\kappa(x)$  can be determined by

$$\kappa(x) = \frac{\varepsilon_1(x) - \varepsilon_2(x)}{h} \tag{1}$$

where  $\varepsilon_1(x)$  and  $\varepsilon_2(x)$  are the strains at the bottom and the top of the cross section at the location *x*, respectively, and *h* is the height of the cross section. Subsequently, the bending stiffness *EI*(*x*) can be calculated:

$$EI(x) = \frac{M(x)}{\kappa(x)}$$
(2)

With the bending moment M(x) and the strains  $\varepsilon_I(x)$  and  $\varepsilon_2(x)$  resulting from the nonlinear FE analyses, the bending stiffness distributions of all scenarios are computed and shown in Figure 3. For simplification, a dimensionless stiffness reduction factor f(x) with  $0 \le f(x) \le 1$  is introduced, which represents the ratio between the computed bending stiffness EI(x) and the original bending stiffness of the uncracked and undamaged cross section  $EI_0$ :

$$f(x) = \frac{EI(x)}{EI_0} \tag{3}$$



Figure 3. Actual stiffness distributions for all scenarios determined from FE analysis.

Based on the stiffness distributions presented in Figure 3, the following conclusions can be drawn on the identifiability of the individual scenarios:

When the force is small (L1), in the undamaged beam B1 slight cracks are formed in the left span, resulting in a reduction in bending stiffness of about 20 % with respect to the bending stiffness of the uncracked und undamaged cross section. At the middle support the beam remains uncracked. Since there is no considerable difference between the bending stiffness distributions of the undamaged beam B1 and the damaged beams B2 and B3, only scenario B1-L1 is selected for structural identification (see next chapter).

For a medium force (L2), the bending stiffness in the left span of the undamaged beam B1 is reduced by about 70 %. At the middle support the stiffness degradation is about 40 %. The damage to the lower reinforcement in B2 causes a reduction in bending stiffness in the left span by about 80 %. As a result, a redistribution of the bending moment occurs, which leads to a reduction in bending stiffness by 80 % at the middle support. The bending stiffness distribution of beam B3 is almost equal to

the one of beam B1. Hence, only the scenarios B1-L2 and B2-L2 are selected for structural identification (see next chapter).

When the undamaged beam B1 is loaded by a large force (L3), the bending stiffness in an extended region of the left span decreases by 80 %, and at the middle support by 85 %. For the beam B2, the bending stiffness distribution is similar to that of beam B1, except a small additional reduction in the region of the damage by about 10 %. The bending stiffness distribution in the left span of beam B3 is similar to the one of beam B1. The bending stiffness of beam B3 at the middle support decreases dramatically by nearly 100 %, indicating that a plastic hinge has been formed. All scenarios B1-L3, B2-L3 and B2-L3 are selected for structural identification (see next chapter).

#### **STEP 2 – STRUCTURAL IDENTIFICATION**

In this step, the system and the load properties of the previously selected scenarios should be identified by FE model-updating. In the identification process the FE models are analyzed by assuming linear elastic structural behavior, that means without considering bending stiffness degradation due to cracking of the concrete. Instead, bending stiffness degradation is described by establishing stiffness reduction functions: The bending stiffness EI of a finite element whose center of gravity is at the location x is multiplied with the value f(x) of the stiffness reduction function. In this paper piecewise linear stiffness reduction functions f(x) are used. For this, the beam is divided into 12 subsections (6 subsections per span) with a length of 400 mm each. In every subsection, a linear stiffness reduction function is established, which yields a dimensionless value with  $0 < f(x) \le 1$  (where 1 means the section is completely uncracked and 0 means complete failure). To calibrate FE models of the beams, a FE baseline model is developed which is parameterized in terms of the unknown parameters which have to be identified. In total the baseline model comprises a number of 12 updating parameters, namely the functional values  $f_1, f_2, f_3, \dots, f_{11}$  of the stiffness reduction function f(x) and the location of the single force  $e_F$  (see Figure 4). The magnitude of the single force is computed from the measured reaction forces.



Figure 4. Updating parameters of the baseline model.

#### **RESULTS AND DISCUSSION**

The following diagrams (Figure 5) show the stiffness distribution along the beam resulting from the stiffness reduction functions which had been identified during the model-updating process of the previously selected scenarios. To evaluate the quality of the identified parameters, the actual stiffness distribution of the respective scenario is also included in the diagrams. The identification results of the location of the load is not displayed; however the load was identified with an accuracy of 96 % up to 100 %.

For all identified stiffness distributions, the difference between the predicted responses of the model and the observed responses is between 1 and 6%. In contrast to that, the difference between the identified and the actual values of the stiffness distribution is considerably larger and ranges from 0 to 30%. In the center of the left span and at the middle support, the stiffness values had been identified with sufficient accuracy. In general, a reasonably good agreement had been achieved between the identified and the actual stiffness distributions. However, in some regions the error in the identified stiffness distribution is relatively large, for example for B1-L3, B2-L2 and B2-L3 at x=1600 mm. The reason for this is the fact, that the applied stiffness reduction function is not capable to approximate steep curves. Furthermore, in the right span errors of up to 30% occur. Probably these errors are present, because by this the inaccuracy of the identified stiffness distribution in the left span is compensated, so that a better agreement between the predicted and observed responses is achieved in total.



![](_page_6_Figure_4.jpeg)

#### CONCLUSIONS

In this paper, a model-updating approach for long-term SHM of RC bridges based on describing cracked or damaged regions of a structure by applying piecewise linear stiffness reduction functions is presented. During the model-updating approach, the unknown parameters of these functions as well as the load properties are identified by minimizing the difference between static responses of the model and the corresponding responses observed on the real structure. By means of numerical simulations it had been evaluated, whether the approach can be used to distinguish between stiffness degradation resulting from cracking of the concrete and damages to the reinforcement.

From the results the following conclusions can be drawn: (1) Although the severity of the damage introduced into the beam (25 % reduction in the cross sectional area of the reinforcement) is quite large, it has only minor effect on the bending stiffness of the structure; in some cases, no considerable difference between the stiffness of the undamaged and the damaged beams is observable. (2) Although the difference between the predicted and observed responses is small ( $\leq 6$  %) for all considered scenarios, the difference between the identified and the actual bending stiffness is up to 30 %. Therefore the fine stiffness degradations due to the introduced damages could not be detected. However, the bending stiffness distributions of the beams had basically been identified. A further improvement of the accuracy can be achieved by applying stiffness reduction functions with a better approximation capability.

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