Damage Identification from Power Spectrum Density Transmissibility

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ABSTRACT

Damage identification under real operating conditions of the structure during its daily use would be suitable and attractive to civil engineers due to the difficulty and problems of carrying out controlled forced excitation tests on this kind of structures. In this case, output-only response measurements would be available, and an output-only damage identification procedure should be implemented. Transmissibility, defined on an output-to-output relationship, is getting increased attention in damage detection applications because of its dependence with output-only data and its sensitivity to local structural changes. In this paper, a method based on the power spectrum density transmissibility (PSDT) is proposed to detect structural damage.

INTRODUCTION

Periodic inspection and maintenance of structures are essentials for the purpose of ensuring their healthy operational condition. Many methods have been proposed in the last years for the detection and location of damage in structural systems [1, 2]. These methods include time and frequency domain techniques and empirical and model-based approaches. The key point for most of the available techniques is the comparison between features obtained from experimental response measurements and features evaluated under normal working conditions.

Modal testing and modal parameter identification have been one core issue in dynamics-based structural health monitoring. From input and output measurement data, modal parameters can be obtained through the frequency response functions

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(FRF). However, damage identification under real operating conditions of the structure during its daily use would be suitable and attractive to civil engineers due to the difficulty and problems of carrying out controlled forced excitation tests on this kind of structures. In this case, output-only response measurements would be available, and an output-only damage identification procedure should be implemented.

Classical output-only techniques often require the operational forces to be white noise. This is not necessary for the proposed transmissibility-based approach [3, 4, 5]. The unknown operational forces can be arbitrary as long as they are persistently exciting in the frequency band of interest. The transmissibility function, defined as the frequency-domain ratio between two outputs, describes the relative admittance between the two measurements and makes possible the damage detection without any assumption about the nature of the excitations although different loading conditions have to be obtained during the experiments. The scalar transmissibility is deterministic in case of one single operational force. However, when several (uncorrelated) operational forces are exciting the structure, the scalar transmissibility is in general not deterministic anymore and the concept of multivariable transmissibility [6] is introduced increasing the complexity of the problem.

In practice, a transmissibility measurement can be estimated in several ways although the most common choice is using an estimator involving cross-and auto-power spectral density functions of the output responses. Furthermore, any measurable output, such as strain, displacement, acceleration, might be used to evaluate the spectral density functions.

In this paper the power spectrum density transmissibility (PSDT) has been used for the damage detection procedure [7]. PSDTs are independent of the applied excitations and transferring outputs at the system poles. The outputs from only one load condition are needed to define the transmissibility and therefore to detect damage circumventing the problems of multiple excitations in large civil engineering structures.

POWER SPECTRUM DENSITY TRANSMISSIBILITY (PSDT)

Transmissibility measurement is a output-only technique, very suitable therefore for operational dynamic analysis. Transmissibility functions are defined by taking the ratio of two response spectra by assuming a single force applied in an input degree of freedom. When several operational forces are exciting the structure, the calculation of the transmissibility becomes much more complex. The use of the power spectrum density transmissibility, estimated by using a reference response signal instead of an excitation signal, allows avoiding this problem.

The power spectrum density transmissibility between the outputs $y_i(t)$ and $y_j(t)$ with reference to the output $y_p(t)$ is defined as the ratio between the power spectral densities responses $X_i^p(\omega)$ and $X_j^p(\omega)$:

$$T_{ij}^p(\omega) = \frac{X_i^p(\omega)}{X_j^p(\omega)} \tag{1}$$

where ω is the frequency.

Transmissibility can be measured in several ways, being one of the most common the use of cross- and auto-power functions G:

$$\left|T_{ij}^{p}(\omega)\right| = \frac{X_i^p(\omega)}{X_j^p(\omega)} = \sqrt{\frac{X_i(\omega)X_p(\omega)}{X_j(\omega)X_p(\omega)}} = \sqrt{\frac{G_{Y_iY_p}(\omega)}{G_{Y_jY_p}(\omega)}} \tag{2}$$

Furthermore, as has been proved [3, 4], when the Laplace variable s approaches system's *r*th pole, denoted by λ_r, the following equation is verified

$$\lim_{s\to\lambda_r} T_{ij}^p(s) = \frac{\phi_{ir}}{\phi_{jr}} \tag{5}$$

Therefore, if two different reference points, k and l, are considered the subtraction of the two PSDTs satisfies

$$\lim_{s\to\lambda_r}(T_{ij}^k(s) - T_{ij}^l(s)) = \frac{\phi_{ir}}{\phi_{jr}} - \frac{\phi_{ir}}{\phi_{jr}} = 0 \tag{6}$$

This means that the system's poles are zeros of the rational function:

$$\Delta T_{ij}^{kl}(s) = T_{ij}^k(s) - T_{ij}^l(s) \tag{7}$$

And poles of its inverse,

$$\Delta^{-1}T_{ij}^{kl}(s) = \frac{1}{\Delta T_{ij}^{kl}(s)} = \frac{1}{T_{ij}^k(s) - T_{ij}^l(s)} \tag{8}$$

The above theoretical results conclude that transmissibility is feasible to establish a rational function $\Delta^{-1}T_{ij}^{kl}(s)$, with poles equal to the system's poles.

DAMAGE DETECTION BY USING PSDT

As shown in the previous section, the natural frequencies can be simply determined from the observation of the peaks in the graphs of the inverse transmissibility subtraction function ITSF, $\Delta^{-1}T_{ij}^{kl}(s)$. Furthermore, these peaks are related with values of the mode shape ratios, i.e. the values of $\Delta^{-1}T_{ij}^{kl}(s)$ at the system poles are related to the scalar operational mode-shape values ϕ_{iv} and. ϕ_{jv} Therefore, once the resonant frequencies are identified, it is also possible to identify vectors from different ITSFs. By choosing a fixed reference DOF j the full unscaled vector for the υth frequency $\left(\Delta^{-1}T_{1j}^{\upsilon}, \Delta^{-1}T_{2j}^{\upsilon}, \cdots, \Delta^{-1}T_{jj}^{\upsilon}, \cdots, \Delta^{-1}T_{Kj}^{\upsilon}\right)$ (K is the number of measured output DOFs) can be constructed from ITSF. The different components of the vector might be defined in the following way:

$$\Delta^{-1}T_{ij}^{\upsilon} = \int_{f_{\upsilon 1}}^{f_{\upsilon 2}} \Delta^{-1}T_{ij}(f)df \tag{9}$$

where $f_{\upsilon 2} - f_{\upsilon 1}$ is the integrated frequency bandwidth for the υth ITSF.

The same procedure should be repeated for each $\Delta^{-1}T_{ij}^{\upsilon}$ by choosing the appropriate bandwidth affecting each system's pole υ.

In particular, the modal assurance criterion (MAC) [8], usually employed to indicate the correlation between two sets of mode shapes, might be used to estimate the correlation between the undamaged ITSFs ($\left\{ITSF_u^{\upsilon}\right\}$) and damaged ITSFs ($\left\{ITSF_d^{\upsilon}\right\}$)

for each mode υ at the measurement degrees of freedom. The new criterion, denoted as *MACITSF*, is for each mode υ as follows:

$$MACITSF_{\upsilon} = \frac{\left| \left\{ITSF_{u}^{\upsilon}\right\}\left\{ITSF_{d}^{\upsilon}\right\}^{T} \right|^{2}}{\left(\left\{ITSF_{u}^{\upsilon}\right\}\left\{ITSF_{u}^{\upsilon}\right\}^{T}\right)\left(\left\{ITSF_{d}^{\upsilon}\right\}\left\{ITSF_{d}^{\upsilon}\right\}^{T}\right)} \tag{10}$$

This index, applied to each υth paired mode, has the advantage that each term is between 0 and 1. An index equal to zero means no correlation between the two sets, while a value equal to one means no change in ITSF, and therefore, no structural damage.

The criterion above has been defined for only one vibration mode υ. To consider the N_m vibrating modes, a total modal assurance criterion has been defined in the following way:

$$MACITSF = \prod_{\upsilon=1}^{Nm} MACITSF_{\upsilon} \tag{11}$$

and an average value might be expressed as follows:

$$AMACITSF = MACITSF \,/\, Nm \tag{12}$$

NUMERICAL EXAMPLE

Numerical simulation is carried out with a cantilever beam which was adopted to examine the performance of the proposed damage detection index. A schematic diagram of this beam with its geometric dimensions and material properties is shown in Fig. 1. Two different simulations were carried out by considering a mesh of 50 beam elements. The beam was assumed to be lightly damped with a constant damping ratio of 0.5%.

The damage was numerically simulated by introducing a stiffness reduction into the elements chosen to be damaged. To excite the undamaged and damaged beams, a simulated impulse force spectrum with constant amplitude of 1000 was applied to the midspan node (node 26) (Fig. 1). Then, the vertical acceleration response PSD of each node was captured and analyzed to construct the cross power spectrum and power spectrum density transmissibility (PSDT) of the first five flexural modes.

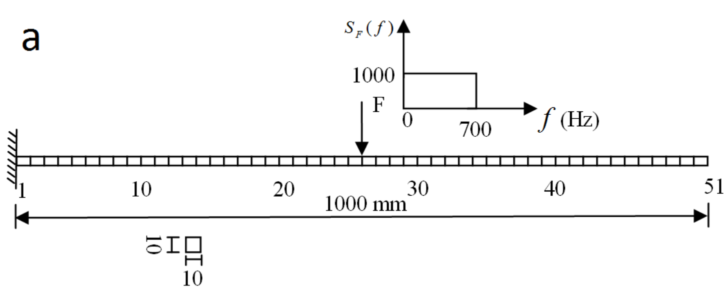

Figure 1. Numerical simulation with 50 elements.

Fig .2 shows the inverse transmissibility subtraction function defined for two nodes randomly selected.

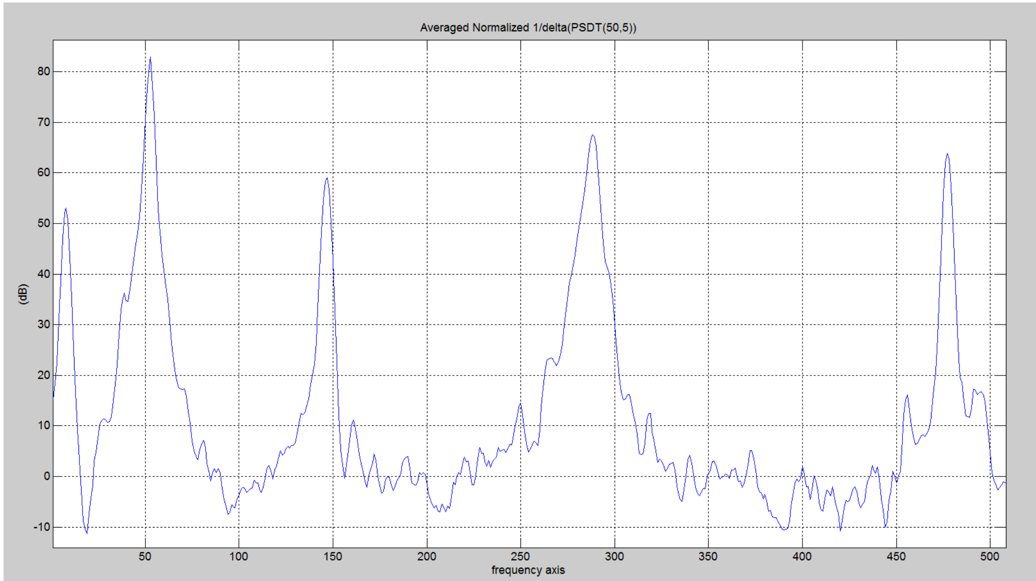

Figure 2. Inverse transmissibility subtraction function.

Firstly, a single damage scenario was considered. Element between nodes 25 and 26 was considered to be damaged and different levels of damage were introduced to study the sensitivity of the proposed method to the damage severity. Predictions using the AMACITSF index are shown in Fig.3. In general, higher levels of damage (lower flexural stiffness) give an AMACITSF value further from one.

Fig.4 shows the AMACITSF value in a multiple damage scenario (three damaged elements). The same scenario is studied by introducing a 2% noise and results are shown in Fig.5.

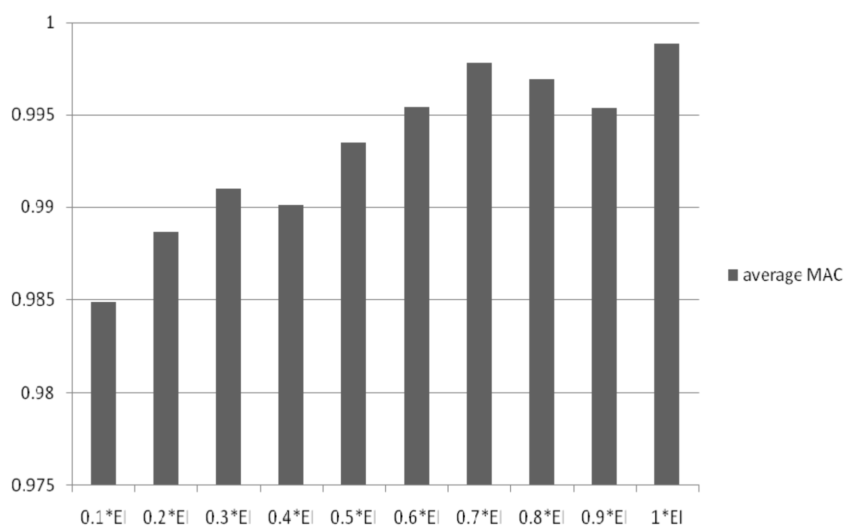

Figure 3. AMACITSF value in a single damage scenario.

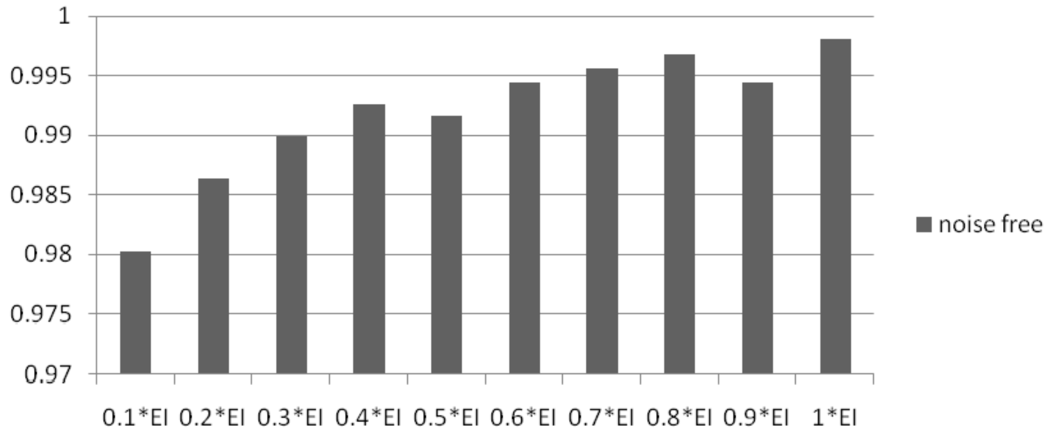

Figure 4. AMACITSF values in a multiple damage scenario.

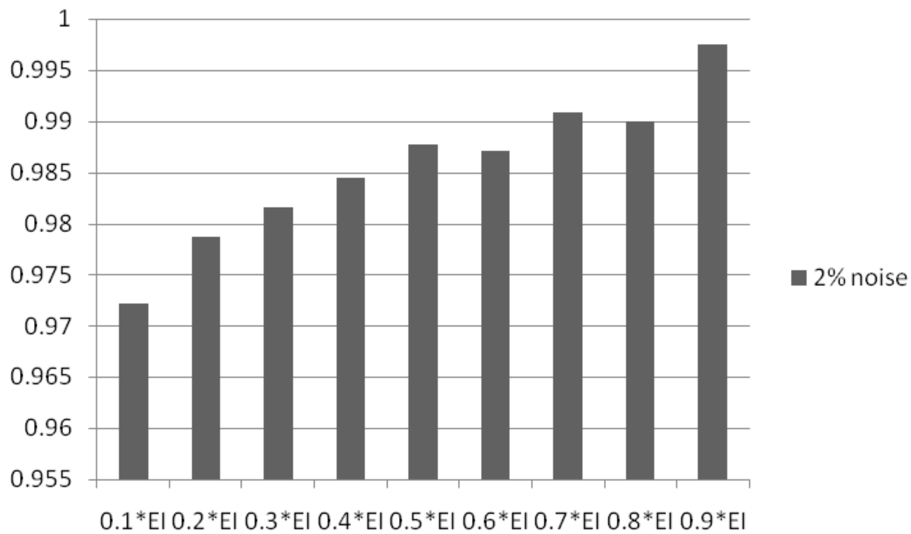

Figure 5. AMACITSF values in a multiple damage scenario with 2% noise.

CONCLUSIONS

One damage detection procedure based on the power spectral density transmissibility has been proposed in this paper. The ability of detecting minor damage qualifies the proposed method for a possible application on real structures under random excitation. New improvements of the method, such as the ability of locating damage, should be performed in the future.

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