Experimental Study of a Model-Free Method for Identification of Stiffness-Related Structural Damages

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ABSTRACT[1]

This paper presents a theoretical derivation and an experimental verification of a model-free method for identification of stiffness-related damages. The proposed method requires no parametric numerical model of the monitored structure, which obviates the need for initial model updating and fine tuning. The paper introduces the general methodology, including the inverse problem, focuses it on stiffness-related damages, and reports on an experimental verification. A 4-meter-long, 70-element truss steel structure made of a commercially available system of nodes and connecting tubes is used for that purpose. Damage is simulated by an intentional replacement of a structural element.

INTRODUCTION

The long-term motivation behind this research is the need for a practical SHM technique that could be used in black-box type monitoring systems. Real-world structures are large and complex. It is usually difficult to monitor them globally using a model-based approach [1-5], since such approaches require a parametric model of the monitored structure (like finite element models), and such models are in practice difficult to update accurately enough. A possible solution are pattern recognition methods, as they require no parametric numerical structural models [6,7]. However, such approaches are usually capable only of damage detection, even if sometimes they offer also limited damage localization possibilities via physical distribution of available sensors across the structure [8,9].

The model-free methodology [10] developed in this paper aims at addressing the mentioned deficiencies of model-based as well as pattern recognition methods:

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1) The approach requires no parametric numerical model of the monitored structure. Thus, there is no need for preliminary model updating and tuning, which is a crucial and usually difficult step in model-based approaches.
2) The approach uses a non-parametric model of the structure, which is directly based on experimentally measured quasi impulse responses of the monitored structure. Thus, the approach can offer damage localization and quantification, besides pure detection. An effective sensitivity analysis is also possible, so that quickly convergent second-order optimization methods can be used for effective damage identification.

The task of damage identification is formulated in the form of an optimization problem of minimizing the discrepancy between the measured and the modeled time-domain structural responses. The virtual distortion method (VDM [11]) is used, as it allows the structure to be modeled in an essentially non-parametric way via a set of its experimentally obtained quasi impulse responses, reduced to the degrees of freedom (DOFs) related to the potential modification points. Structural modifications are modeled using certain pseudo loads that are imposed on the undamaged structure. Their effect on the structural response is expressed in the form of a convolution with the experimentally obtained quasi impulse responses of the unaffected structure. The pseudo loads that properly model given damage are found by solving certain linear integral equation. A related formulation in frequency domain can be found in [12].

The paper first introduces the general methodology, and then focuses it on stiffness-related damages. Finally, it reports on an experimental verification using a 4-meter-long, 70-element steel truss structure made of a commercially available system of nodes and connecting tubes. Damage of an element is simulated by replacing it with an element of a similar weight but made of aluminum instead of steel.

THE DIRECT PROBLEM – GENERAL METHODOLOGY

Assume that the original intact structure satisfies the standard equation of motion:

$$\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{C}\dot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{K}\mathbf{u}^{\mathrm{L}}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{f}(t)$ is a given excitation vector. Assume also that structural modifications and damages can be quantified in terms of certain modifications to structural mass and stiffness matrices $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$ respectively. As a result, the modified/damaged structure is assumed to satisfy the following equation of motion:

$$(\mathbf{M} + \Delta\mathbf{M})\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + (\mathbf{K} + \Delta\mathbf{K})\mathbf{u}(t) = \mathbf{f}(t), \tag{2}$$

where $\mathbf{u}(t)$ is used to denote the response of the modified/damaged structure to the same excitation $\mathbf{f}(t)$.

According to the basic idea of the virtual distortion method (VDM, [11]), the damages and modifications can be equivalently modeled by imposing a certain vector $\mathbf{p}^0(t)$ of pseudo loads onto the original undamaged structure. By moving in (2) the modification terms to the right-hand side, one can obtain

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{p}^0(t), \tag{3}$$

where the pseudo load vector is implicitly defined as

$$\mathbf{p}^0(t) = -\Delta\mathbf{M}\ddot{\mathbf{u}}(t) - \Delta\mathbf{K}\mathbf{u}(t). \tag{4}$$

Exact impulse responses

Notice that (3) is in fact the equation of motion of the original intact structure subjected to the pseudo load $\mathbf{p}^0(t)$ besides the testing excitation $\mathbf{f}(t)$. Therefore, the response $\mathbf{u}(t)$ of the *modified/damaged* structure can be expressed in terms of the impulse response matrices $\mathbf{B}^0(t)$ and $\ddot{\mathbf{B}}^0(t)$ of the original *undamaged* structure as

$$\begin{aligned} \mathbf{u}(t) &= \mathbf{u}^{\mathrm{L}}(t) + \int_0^t \mathbf{B}(t-\tau)\mathbf{p}^0(t)\mathrm{d}\tau = \mathbf{u}^{\mathrm{L}}(t) + (\boldsymbol{\mathcal{B}}^0\mathbf{p}^0)(t), \\ \ddot{\mathbf{u}}(t) &= \ddot{\mathbf{u}}^{\mathrm{L}}(t) + \int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}^0(t)\mathrm{d}\tau = \ddot{\mathbf{u}}^{\mathrm{L}}(t) + \left(\ddot{\boldsymbol{\mathcal{B}}}^0\mathbf{p}^0\right)(t), \end{aligned} \tag{5}$$

where the matrices $\mathbf{B}^0(t)$ and $\ddot{\mathbf{B}}^0(t)$ collect respectively the displacement and acceleration impulse responses of the original undamaged structure. Notice that the latter includes an impulsive component at time $t = 0$ in all its entries corresponding to collocated pairs of accelerometers and excitation DOFs. The convolutions with the impulse response matrices can be expressed also in a more concise way using the corresponding matrix integral operators $\boldsymbol{\mathcal{B}}^0$ and $\ddot{\boldsymbol{\mathcal{B}}}^0$.

Substitution of (5) into (4) yields

$$\mathbf{p}^0 + \left(\Delta\mathbf{K}\boldsymbol{\mathcal{B}}^0 + \Delta\mathbf{M}\ddot{\boldsymbol{\mathcal{B}}}^0\right)\mathbf{p}^0 = -\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}} - \Delta\mathbf{K}\mathbf{u}^{\mathrm{L}}, \tag{6}$$

which is a linear integral equation of the Volterra type. If the impulsive components in the acceleration impulse response matrix are taken into account, it can be shown that equation (6) is of the second kind and thus uniquely solvable [13], provided the matrix $\mathbf{M} + \Delta\mathbf{M}$ is positive definite. As the original mass matrix $\mathbf{M}$ can be assumed to be always positive definite, (6) is uniquely solvable for all mass modifications that are small enough. Notice that the impulse responses need to be measured only locally, that is only in the DOFs that are related to the potential modifications/damages, since in other DOFs the pseudo loads vanish, which can be easily seen in (6).

Given the modifications and solved (6), the resulting pseudo load vector is substituted into (5) to obtain the response of the modified/damaged structure.

Experimental quasi impulse responses

The solution outlined so far assumes that the structural impulse responses are measured and available. However, exact impulse responses are hardly available in practice: one can measure only responses to quasi-impulsive excitations that last several time steps. Two solutions are possible: (i) either the measured responses are deconvolved with respect to the actually applied quasi-impulsive excitations in order to obtain the exact impulse responses or (ii) the measured responses are directly used in computations. The first approach requires performing a separate ill-conditioned deconvolution for each pair of a quasi-impulsive excitation and the corresponding response. This is avoided by the second approach, which implicitly embeds the deconvolutions in a modified version of (6). The pseudo loads are expressed in the form of a convolution of the actually applied quasi-impulsive excitations $h_i(t)$, which all have to satisfy $h_i(t) = 0$ for $t \leq 0$, with certain unknown functions $p_i(t)$,

$$p_i^0(\mathrm{t}) = (h_i * p_i)(t) = \int_0^t h_i(t-\tau)p_i(\tau)\mathrm{d}\tau, \tag{7}$$

where i indexes all the DOFs related to the considered modifications/damages. Equation (7) is collected for all the involved DOFs and stated in the operator notation as

$$\mathbf{p}^0 = \boldsymbol{\mathcal{H}}\mathbf{p}, \tag{8}$$

where $\boldsymbol{\mathcal{H}}$ denotes the respective diagonal matrix convolution operator.

Substitution of (8) into (5) and (6) yields

$$\mathbf{u} = \mathbf{u}^{\mathrm{L}} + \boldsymbol{\mathcal{B}}\mathbf{p} \qquad\qquad \ddot{\mathbf{u}} = \ddot{\mathbf{u}}^{\mathrm{L}} + \ddot{\boldsymbol{\mathcal{B}}}\mathbf{p} \tag{9}$$

and

$$\left(\boldsymbol{\mathcal{H}} + \Delta\mathbf{K}\boldsymbol{\mathcal{B}} + \Delta\mathbf{M}\ddot{\boldsymbol{\mathcal{B}}}\right)\mathbf{p} = -\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}} - \Delta\mathbf{K}\mathbf{u}^{\mathrm{L}}, \tag{10}$$

where $\boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{B}}^0$ and $\ddot{\boldsymbol{\mathcal{B}}} = \boldsymbol{\mathcal{H}}\ddot{\boldsymbol{\mathcal{B}}}^0$ are the matrix integral operators that correspond to the convolutions with the experimentally measured responses to the quasi-impulsive excitations $h_i(t)$.

THE DIRECT PROBLEM – STIFFNESS-RELATED DAMAGES

In case when the modeled damages affect only the stiffness of the involved elements, and are thus quantified by $\Delta\mathbf{K}$ only, the derived formulas can be simplified into

$$(\boldsymbol{\mathcal{H}} + \Delta\mathbf{K}\boldsymbol{\mathcal{B}})\mathbf{p} = -\Delta\mathbf{K}\mathbf{u}^{\mathrm{L}} \qquad\qquad \mathbf{u} = \mathbf{u}^{\mathrm{L}} + \boldsymbol{\mathcal{B}}\mathbf{p}. \tag{11}$$

Required data and computations

The model-free approach described here uses a local non-parametric model of the undamaged structure that consists of its (i) the matrix $\mathbf{B}(t)$ of its structural quasi impulse responses, and (ii) the structural response $\mathbf{u}^{\mathrm{L}}(t)$ to the testing excitation $\mathbf{f}(t)$.

These characteristics can be measured experimentally prior to modeling of the damages. According to (10) or (11, left), the pseudo loads vanish in the DOFs that are not directly related to the damages. As a result, the responses that constitute the model need to be measured only by the sensors intended for identification and in the DOFs related to potential damages, which can form only a small subset of all structural DOFs. As a result, full instrumentation of the involved structure is not necessary, which makes experimental measurements more feasible.

Given such a model of the unmodified structure and a modification defined by $\Delta\mathbf{K}$, the response $\mathbf{u}(t)$ of the modified structure to the same testing excitation $\mathbf{f}(t)$ is computed in two steps: (i) the equivalent convolution functions $\mathbf{p}(t)$ are found by solving (11,left) and (ii) the response is computed by (11,right). Notice that all these computations are performed based directly on experimentally measured data, so that there is no need to build and update a parametric numerical model of neither the unmodified nor the modified structure.

THE INVERSE PROBLEM

The inverse problem of identification of stiffness-related damages is formulated here as an optimization problem of minimizing the mean square discrepancy between the measured response $\mathbf{u}^{\mathrm{M}}(t)$ and the modeled response $\mathbf{u}(t)$ of the modified structure,

$$F(\Delta\mathbf{K}) = \frac{1}{2}\int_0^T \|\mathbf{d}(t)\|^2 \mathrm{d}t = \frac{1}{2}\langle \mathbf{d}, \mathbf{d}\rangle, \tag{12}$$

where $\langle\cdot,\cdot\rangle$ is the scalar product, T is the length of the considered time interval and

$$\mathbf{d}(t) = \mathbf{u}^{\mathrm{M}}(t) - \mathbf{u}(t), \tag{13}$$

and which is minimized with respect to a given set of parameters that define the damage $\Delta\mathbf{K}$.

The method of adjoint variable [14] can be used for fast sensitivity analysis. The derivative of F with respect to the αth damage parameter is

$$\frac{\partial F}{\partial \mu_\alpha} = \langle \boldsymbol{\lambda}, \Delta\mathbf{K}_\alpha \mathbf{u}\rangle, \tag{14}$$

where $\Delta\mathbf{K}_\alpha$ denotes the derivative of $\Delta\mathbf{K}$ with respect to the αth optimization variable and $\boldsymbol{\lambda}(t)$ is the vector of the adjoint variables, which is computed as the solution to the following integral equation:

$$(\boldsymbol{\mathcal{H}}^* + \boldsymbol{\mathcal{B}}^*\Delta\mathbf{K})\boldsymbol{\lambda} = \boldsymbol{\mathcal{B}}^*\mathbf{d}, \tag{15}$$

where the superscript $*$ denotes the adjoint operator. In a similar way, see e.g. [10], the second order derivative of the objective function can be computed as

$$\frac{\partial^2 F}{\partial \mu_\alpha \partial \mu_\beta} = \langle \mathbf{u}_\alpha, \mathbf{u}_\beta\rangle + \langle \boldsymbol{\lambda}, \Delta\mathbf{K}_\alpha \mathbf{u}_\beta + \Delta\mathbf{K}_\beta \mathbf{u}_\alpha + \Delta\mathbf{K}_{\alpha\beta}\mathbf{u}\rangle, \tag{16}$$

where the derivatives of the response are computed by differentiating (12),

$$(\boldsymbol{\mathcal{H}} + \Delta\mathbf{K}\boldsymbol{\mathcal{B}})\mathbf{p}_\alpha = -\Delta\mathbf{K}_\alpha \mathbf{u} \qquad\qquad \mathbf{u}_\alpha = \boldsymbol{\mathcal{B}}\mathbf{p}_\alpha. \tag{17}$$

Given the response derivatives, the first derivative of the objective function can be computed for the purpose of verification of (14) also as

$$\frac{\partial F}{\partial \mu_\alpha} = -\langle \mathbf{d}, \mathbf{u}_\alpha\rangle. \tag{18}$$

EXPERIMENTAL VERIFICATION

The setup

A 3D steel truss structure is used in the experimental verification. It is of 4 meter length, 32 kg weight, consists of 70 elements and 26 nodes, see Figure 1. A damage of the marked element (weight 0.375 kg and axial stiffness *EA*=13,850 kN) is simulated by replacing it with an aluminum element of a comparable weight, but significantly reduced stiffness (weight 0.300 kg, axial stiffness *EA*=9,270 kN). A modal hammer is used to generate the quasi impulse response matrix $\mathbf{B}(t)$ and the testing excitation $\mathbf{f}(t)$, which is applied in the *z*-direction in the node shown in Figure 1. The discrepancy function $\mathbf{d}(t)$ is constructed by comparing the displacement responses measured in *z*-direction in another node, see Figure 1.

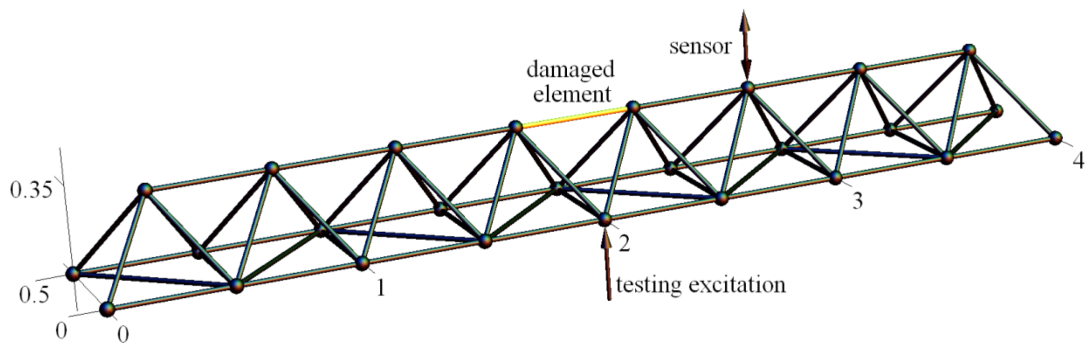

Figure 1. The 3D truss structure used for experimental verification.

Identification results

Figure 2 plots the objective function in dependence on the axial stiffness of the damaged element. In the computations, the mass of the modified element is assumed to remain the same. The minimum of the function is found at 9,560 kN. The identification result slightly overestimates the actual axial stiffness of the modified element (9,270 kN), see Figure 2. This is consistent with the fact that the mass reduction is neglected in computations: the measured and modeled basic vibration periods can be fitted only if a surplus mass is countered with an surplus increase of stiffness.

Figure 3 compares the measured responses of the original and the damaged structures, $\mathbf{u}^{\mathrm{L}}(t)$ and $\mathbf{u}^{\mathrm{M}}(t)$, with the modeled response $\mathbf{u}(t)$ at the identified optimum value of axial stiffness of the damaged element.

Since there is only one damage parameter, the responses are fitted mainly based on the basic vibration period. Figure 4 compares the responses modeled for different levels of axial stiffness of the affected element in order to demonstrate the effect of damage on the basic vibration period. Notice that the assumed level of the axial stiffness affects not only the vibration period, but also its amplitude and/or damping ratio. If it is an actual effect or just a numerical artifact remains a subject of ongoing research.

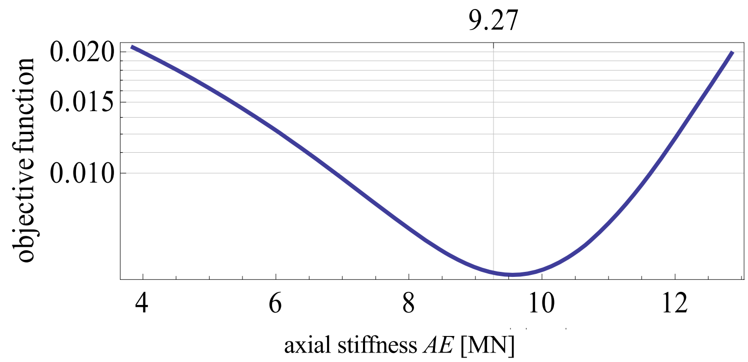

Figure 2. The objective function.

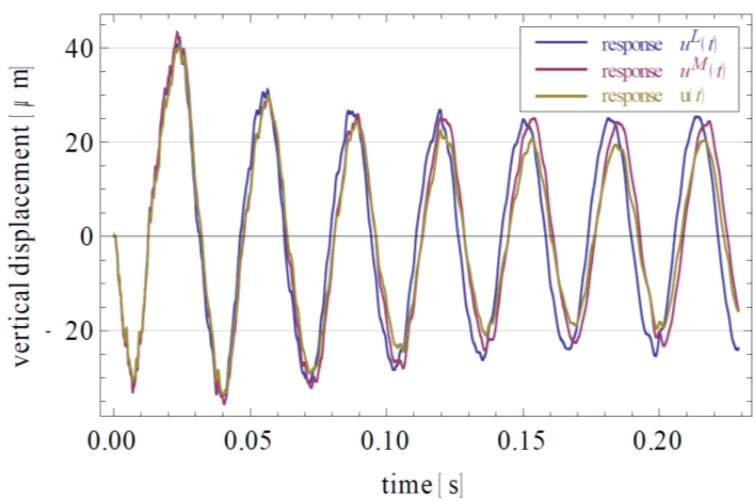

Figure 3. Measured response of the undamaged structure, measured response of the damaged structure, and the optimally modeled response.

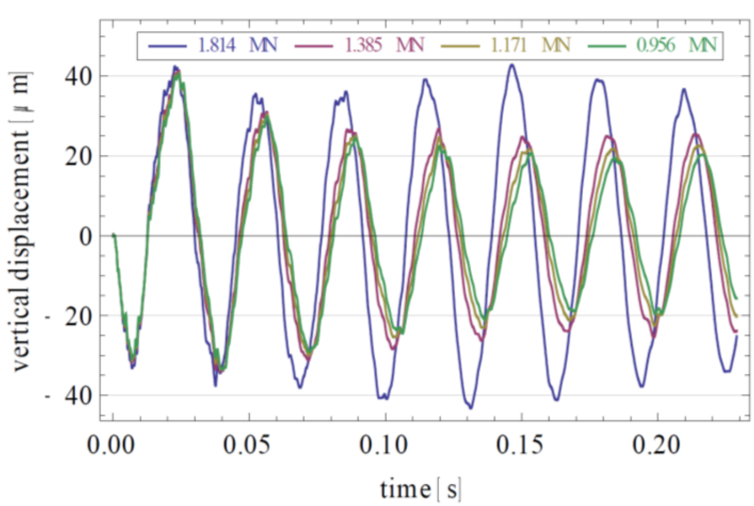

Figure 4. Responses modeled for different assumed values of the axial stiffness of the involved element (1,385 kN is the basic measured response $\mathbf{u}^L(t)$, which is modified by pseudo loads).

CONCLUSION

This paper develops and experimentally verifies a model-free approach to identification of stiffness-related damages. The approach is based on the virtual distortion method (VDM), and uses an essentially non-parametric purely experimental model of the monitored structure. No parametric numerical model, such as a FE model, is required, which in many application is an advantage as allows to avoid the error-prone stage of model updating.

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