ABSTRACT

In this paper we present results of the research aimed at designing and manufacturing a prototype of a SHM system that employs Phased Arrays (PA) for generating and receiving Lamb Waves (LW). A linear array was considered first but it appeared that an unequivocal localization of damage on a plane panel requires a 2D array’s topology.

In this paper we present a new method for theoretical, numerical and experimental investigations of various 2D arrays’ topologies for SHM of planar structures. The theoretical evaluation is performed using the frequency-dependent transfer function that affects propagation of Lamb waves (LWs) through the dispersive medium and enables investigation of the arrays’ performance for a defined excitation signal. The numerical simulations are conducted using local interaction simulation approach (LISA) implemented on the NVIDIA® CUDA® graphical processing unit (GPU), which considerably accelerates 3D simulations of LWs propagation in a short time period. Finally, scanning laser vibrometer is used to sense the LWs excited by PZT transducers, in multiple points corresponding to the locations of the 2D array elements. In this way performance of various array topologies can be evaluated experimentally in the reception mode without the need of physical prototype - a change of topology requires only straightforward modification of the measurement points’ distribution at the tested plate.
INTRODUCTION

Lamb waves (LW) based methods can be used for structural health monitoring (SHM) of plate-like structures that require high safety standards. Lamb waves (LW) based methods can be used for structural health monitoring (SHM) of plate-like structures that require high safety standards. Phased arrays (PA) with multiple, densely spaced transducers, enable effective beamforming offering a superior signal to noise ratio, which makes possible assessment of large panels from a single, fixed position. A serious disadvantage of linear arrays applied to SHM of planar structures, which prevents unequivocal damage localization, results from the mirrored images that they produce. 2D arrays do not have this drawback and they even are capable of mode selectivity when generating and receiving LWs. Performance of 2D arrays depends on their topology as well as the number of elements (transducers) used, and their spacing in terms of wavelength. Therefore SHM systems based on LW excited and sensed by the means of PA are of great interest and a considerable work has been reported in this field [1] concerning different shapes of such arrays. For instance, star [2], square [3], circular [4] and spiral [5] shaped arrays were reported in the literature.

In this paper we present a simple experimental method for evaluation of beampatterns obtained using different topologies of 2D PAs in the case of dispersive signals.

This paper is organized as follows. A brief theoretical overview of PA technique will be given in the first section, and then method of analytical, numerical and experimental dispersive signals evaluation will be outlined. Finally, the proposed method will be used to evaluate the performance of prototypical arrays consisting of 32 sensors.

THEORETICAL BACKGROUND

In the field of SHM a phased array of sensors can be considered as a spatial filter enabling isolation of waves arriving from a desired direction. The filtering operation, however, is almost never perfect, especially, in the case of multimodal dispersive waves that can cause a significant interference. Therefore, an investigation of array’s angular performance is required in the designing process.

Beampattern plot, which is a diagram of power output from a steered array, is used here to illustrate array’s performance. In this paper we present the method to predict the directional characteristics of 2D array that does not require building its prototype. The method consists of 2 steps: in the first step, a set of snapshots scattered by a reflector in the array’s far-field is obtained using an analytical, numerical or experimental approach. In the second step, the acquired signals are processed using the delay-and-sum (DAS) beamformer, which means that the signals are time-shifted prior to summation. The time-shifts are calculated for the desired range of angles using fixed wavenumber. The maximum value of the sum of the signals is then used to form the directivity characteristics. To obtain the beampattern for a wave impinging from another direction, with respect to the array, the reflector can be shifted to another position or the array can be rotated. Since the contactless measurement technique, used in this work, enables easy modification of the measurement points, rotation of the sensing array will be used here.
EVALUATION OF DISPERSIVE SIGNALS

The dispersive snapshots that are the input for DAS beamformer can be acquired in a straightforward manner by measuring structure’s response. In some cases, however, those signals can be obtained theoretically or using numerical simulations. In this section, we will illustrate using examples how different methods can be applied.

Analytical evaluation of dispersive signals

When the dispersion curves are available it is possible to evaluate the response of the structure $V_r(t)$ at the point $x$ due to the excitation $V_e(t)$ [6]. According to the setup presented in Figure 1 the signal reflected from the scatterer $R$ and then captured by the sensor $S_{ion}$ has propagated over the distance $x$ equal to the sum of distances $|AR|$ and $|RS_{ion}|$. In the evaluation presented below a tone-burst signal consisting of 3 cycles of sine at the frequency of 100 kHz modulated with Hanning window was used as an excitation. The same exciting signal was used in all numerical and real experiments. In our example an aluminum plate with thickness 2 mm was used and propagation of a single $A_0$ mode of the wavelength of 12.77 mm was assumed.

![Figure 1. Example of the setup used to acquire signals scattered by reflector R using a 2D array.](image)

Numerical evaluation of dispersive signals

The simulation of wave propagation phenomena was performed using the local interaction simulation approach with sharp interface model (LISA/SIM) [7]. The algorithm is well suited for parallel processing and it has been implemented using graphical processing units with compute unified device architecture (CUDA) technology for the wave propagation calculations [8].

In the numerical simulations for our example a model of an aluminum plate with the size of 700x700x2 mm was built. The plate was modeled using a high resolution 3D mesh with cell size of $c = 0.5mm$. The reflector $R$ in the setup presented in Figure 1 was modeled as a round hole. The center node of the plate’s model was used as an
actuator exciting the tone-burst signal. The out-of-plane displacement of the plate model due to this excitation can be seen in Figure 2a.

Three different PAs were considered with sensor topologies presented in Figure 3 and their rotated equivalents were used in the signal acquisition. Therefore, only one simulation was required to obtain the beampatterns of the arrays at various angles. Out-of-plane displacements of the nodes corresponding to the locations of the arrays’ receivers were captured and considered as the input signals for DAS.

![Image of simulation and setup](image)

**Figure 2.** Out-of-plane displacement obtained using LISA simulation of the $A_0$ wave excited by tone-burst a). Experimental setup used for the vibrometer measurements b).

**Experimental evaluation**

Laser vibrometer can also be used to facilitate 2D array design. Vibrometer can experimentally evaluate beampatterns of the considered array topologies realized as a virtual arrays. The beampattern for a wave impinging from different directions can be easily obtained by shifting the reflector to respective positions or the rotating the virtual sensing array, since the presented method involves contactless measurement with the use of a laser scanning vibrometer [9].

A series of experiments was conducted using the setup presented in Figure 2b to validate the numerical methods for beampatterns investigation presented above. An aluminum plate of the size of 1000x1000x2 mm was instrumented with a PZT actuator CMAP12 from Noliac, Denmark. As a signal generator PAQ/PAS from EC Electronics, Poland, was used to excite a tone-burst signal: 3 cycles of sine at the frequency of 100 kHz modulated with Hanning window. An additional mass was coupled to the plate to produce the reflections. The response of the structure was sensed using the scanning laser vibrometer PSV-400 from Polytec, Germany. An example of the measurement grid points displayed on the plate is shown in Figure 3d-f. The experiments were repeated for all topologies presented in Figure 3a-c and their equivalents rotated with steps of 10° in the azimuth range of 0°-90°.
SHAPES OF THE EVALUATED ARRAYS

The method presented in this paper can be used to evaluate any shape of a 2D array, however, only the topologies presented in Figure 3 will be discussed in this work. All analyzed arrays consist of 32 sensors. The star-shaped array, presented in Figure 3a consists of 4 linear sub-arrays intersecting at an angle of 45°. The elements of the sub-arrays are spaced at a distance $d = 5$ mm, therefore the outside diameter of the array was 40 mm. The circular-shaped topology shown in Figure 3b was made up of 2 concentric circles. The diameter of the bigger one was 25.62 mm and the distance denoted $d$ in the figure is equal to 5 mm. Finally, the spiral-shaped array, presented in Figure 3c was created in the way described in [5]. The transducers are placed on concentric circles with radii increasing with a step of 5 mm, 4 sensors are placed in each circle, which is rotated with an angle of 15° with respect to the former one. The spacing of the subsequent sensors varied from 5.11 to 8.87 mm and the outside diameter of the array is 80 mm.

Figure 3. Investigated array topologies: star-shaped a), circular b), spiral c). Laser measurement points corresponding to the respective array topologies.

INFLUENCE OF BEAMPATTERN ON DAMAGE-IMAGING

DAS beamforming was applied to the acquired signals in the second step of the proposed method. DAS processing of the signals resulted in the images seen in Figure 4a-c that show the sum of the delayed signals captured in the points corresponding to the star, circular and spiral-shaped virtual arrays, respectively. The maximum values existing at the successive azimuths were analyzed and are presented as polar plots in Figure 4d-f, respectively.
Figure 4. Damage images obtained with the signals acquired with the laser vibrometer using star-shaped a), circular b) spiral d) virtual sensing arrays. Beampatterns obtained from the peak values extracted for successive azimuths.

COMPARISON OF THE INVESTIGATED BEAMPATTERNS

The method presented above was used to process the theoretical, numerical and experimental signals. An example of the beampatterns obtained using this procedure can be seen in Figure 5a–c for the wave with an arbitrary selected incident angle of 60°, for spiral, circular and star-shaped array, respectively. Good agreement of the directivity characteristics obtained theoretically, numerically and experimentally can be observed for all of the topologies, however, the experimental beampatterns are better reproduced by the simulated than the theoretical ones. It can be explained by the fact that LISA is a more accurate method for modeling of Lamb waves propagation than the theoretical approach based on the simplified transfer function model.
The beampatterns obtained for the waves with incident angle in the range of 0° − 90° were analyzed and parameters estimated from the characteristics, are presented in Table 1. Width of the main lobe was estimated as indicated in Figure 5d, i.e. as its angular width at the level of -3 dB. The maximal and minimal values of lobe width as well as the highest and the lowest sidelobe levels are presented in Table 1. Comparing the values in Table 1 it can be observed that the spiral array has the narrowest main lobe among the investigated topologies, which was expected since this array has the largest aperture.

Table 1. Selected properties of the obtained beampatterns.

<table>
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<tr>
<th>shape</th>
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<th>max main beam width</th>
<th>min main beam width</th>
<th>max side lobe level</th>
<th>min side lobe level</th>
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CONCLUSIONS

In the paper a method for rapid performance evaluation of array topology planar SHM structures was presented. It was shown that directivity characteristics of a 2D array can be evaluated using theoretical or numerical methods that take into account dispersive nature of Lamb waves as well as the tone-burst excitation and can therefore
produce results in a good agreement with the experimental ones. It was also shown how laser scanning vibrometers, equipped with high frequency cards that have become available in many research laboratories, can be used for experimental verification of the beampatterns of 2D arrays and in this way facilitate their design.

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REFERENCES