

# **Evaluation of an Intrinsic Error Estimator for the Data Fusion of NDT Techniques Used to Identify the Material and Damage Properties of Concrete Structures**

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# ABSTRACT

In this paper, we propose an intrinsic error estimator for the data fusion of NDT techniques used to identify the material and damage properties of concrete structures. This error estimator is chosen based on the global distribution of the data fusion in the space of the identified material properties. The main idea is to evaluate the accuracy of the result in estimating the gap between the most and the worst likely solutions. This error estimator is applied to synthetic data depending on the parameters of the data fusion such as the regression laws that linked the material properties to the NDT measurements.

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# **INTRODUCTION**

Non-destructive testing (NDT) techniques are mainly used in order to obtain relevant information about material properties and damage states of concrete structures. Nevertheless, the sensivity of these NDT techniques to many unknowns of the material parameters and to the experimental conditions is a major issue that makes difficult to extract a reliable and accurate diagnosis. In order to overcome those limitations a first step is to combine the measurements from multiple techniques as proposed in

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[1, 2, 3]. In [4], a method is proposed to take into account the variability of measurements, and to evaluate the relevance of the NDT techniques using statistical analysis. This method aims at enhancing the diagnosis quality of RC structures.

In this process, we proposed a data fusion methodology that uses multiple NDT techniques [5]. This methodology is based on the possibility theory and allows to post-process different kinds of information from different NDT techniques having some uncertainties involved. In that way, the complementarity of the NDT techniques is used and previous works showed significant improvement in the estimation of mechanical and damage properties such as the compressive strength [6]. Until now, destructive testing has been used in order to evaluate the reliability of such results, but this testing should be reduced or avoided in order to obtain industrial applications. At this point, the current issue is to define an intrinsic error estimator associated with the data fusion in order to enhance the diagnosis quality and the reliability of the results.

## **DATA FUSION PROCESS**

#### Relationship between the observations and the indicators

The principle of this method is to consider simplified relationships between the material properties, called the indicators, and the NDT measurements called the observations. Although the parameters of these relationships are modified with the different compositions of concrete, we then assume that the convergence of the NDT techniques associated with some specific material properties allows to update these relationships and to obtain an adequate result with the inspected concrete. An initial database is therefore used to evaluate the indicators and this database is then updated in order to obtain the right estimation of the indicators.

The data fusion process we implemented is precisely described in [5] and is based on the possibility theory. For each of the observations, we expect a relationship between this observation and the whole indicators. Until now, we have chosen linear functions based on some experimental regression laws but there is no restriction to implement more complex modeling. Ultrasound velocity, radar amplitudes and frequencies, resistivity and electrical capacity of the concrete compose the observations. The relationship between the  $i^{th}$  observation and the indicators  $(I_k)_{1 \le k \le m}$  is expressed in eq. (1) and illustrated for the ultrasound velocity ,  $v_{US}$ , as a function of S and P respectively the saturation rate and the porosity of the concrete :

$$\forall 1 \le i \le p$$
,  $O_i = \sum_{i=1}^m a_k^i I_k$ ,  $v_{US} = 2.9350 \times S - 58.179 \times P + 2736.2$  (1)

## **Data fusion process**

Let's now consider the measurement case  $(O_i^{mes})_{1 \le i \le m}$  associated with the  $(O_i)_{1 \le i \le m}$  observations looking for the  $(I_k)_{1 \le k \le p}$  indicators. For each of the observations, a possibility distribution is obtained using the relationship between the observations and

the indicators. For a simpler illustration, we only consider a two-indicators identification,  $I_1$  and  $I_2$ , using from three to six observations. In that case, the distribution of the  $i^{th}$  observation is associated with the line  $d_i$  in the indicator plan obtained by the following equation :

$$d_{i}: O_{i}^{mes} - a_{1}^{i}I_{1} - a_{2}^{i}I_{2} = 0 \Leftrightarrow d_{i}: O_{i}^{mes} - \cos\left(\beta_{1}^{i}\right)I_{1} - \sin\left(\beta_{2}^{i}\right)I_{2} = 0$$
(2)

The measurement uncertainties are then introduced using a trapezoidal evolution depending on the variability of the corresponding measurement technique as defined in [4] and noted  $\sigma$ .



Figure 1: Illustration of an observation distribution  $\pi(O_i^{mes})$  in the indicator plane.

$$\pi(O_i^{mes}) = \begin{cases} 1 \quad if \quad O_i^{mes} - \cos(\beta_k) \quad I_1 - \sin(\beta_k) \quad I_2 \le \pm 0.2\sigma \\ 1 - \gamma_\sigma \quad if \quad O_i^{mes} - \cos(\beta_k) \quad I_1 - \sin(\beta_k) \quad I_2 \le \pm 2.48\sigma \\ 0 \quad elsewhere \end{cases}$$

The data fusion process now consists in combining all the distributions of the observations using an operator such as :

$$\pi(S) = (1 - \alpha^2) \max_{1 \le i \le m} [t_i \pi(O_i^{mes})] + \alpha^2 \min \left[ \min_{1 \le i \le m} [1 - t_i + t_i \pi(O_i^{mes})], \max_{1 \le i \le m} [\pi(O_i^{mes})] \right]$$

 $\pi(S)$  is the fusioned distribution in the indicator plane and this distribution is normalized. The solution  $(I_1^0, I_2^0)$  is then obtained by the condition  $\pi(I_1^0, I_2^0) = 1$ . Lastly,  $(t_i)_{1 \le i \le m}$  corresponds to the reliability of the  $i^{th}$  observation computed with the regression coefficient of the relationship between the  $i^{th}$ -observation and the indicators.  $\alpha$  is the mean value of all the  $(t_i)_{1 \le i \le m}$ .

# **ERROR ESTIMATOR**

# Issue

We consider two different cases associated with two different relative positions of the observations. The first case shown in figure 2 corresponds to a close-observations configuration. The second case shown in figure 3 corresponds to a spaced-observations configuration. For these both cases, we consider the reference solution associated with the three converging orange lines. Adding some disturbances to one of the observation for each case, the green lines, we note an important change of the data fusion shape of the second case compared to the shape of the reference solution. At the opposite, the data fusion shape of the second case is similar to the shape of the reference solution. Lastly, although the reference and the disturbed distributions of the first case are similar, the gap between their identified indicator values is larger than the gap between the identified indicator values of the second case. From these



Figure 2: Comparison between the reference and the noisy solutions for the closeobservations configuration.



Figure 3: Comparison between the reference and the noisy solutions for the spacedobservations configuration.

results, we firstly propose to choose the observations from the database, based mainly on their relative positions. This choice is related to the evolution of the indicator errors as a function of the observation error. Secondly, we propose to define an intrinsic error estimator based on the converging aspect of the observations. This estimator  $E_{id}$ is expressed as a function of the threshold level  $h_s$  and the area of the distribution  $S_{\pi}$ such as :

$$E_{id} = (1 - \varepsilon) \left( \frac{1 - h_s}{1 - h_s^0} \right) + \varepsilon \frac{S_{\pi}}{S_{\pi}^0} \quad \text{with} \quad S_{\pi} = \iint_{\{(I_1, I_2) \ ; \ \pi(I_1, I_2) = 1\}} dI_1 dI_2 \quad (3)$$

In this expression,  $h_s^0$  and  $S_{\pi}^0$  are reference quantities associated with the reference solution.  $0 < \varepsilon < 1$  is a parameter that emphasizes the influence of  $h_s$  compared to the influence of  $S_{\pi}$ . This definition is justified by the fact that for a given  $h_s$  the best result maximizes  $S_{\pi}$ . Lastly,  $h_s$  is defined by the maximum level of each distribution which corresponds to :

$$h_{s} = \max_{1 \le i \le m} \left[ \left( 1 - \alpha^{2} \right) t_{i} \pi(O_{i}^{mes}) + \alpha^{2} \min \left[ \left[ 1 - t_{i} + t_{i} \pi(O_{i}^{mes}) \right], \pi(O_{i}^{mes}) \right] \right]$$
(4)

#### **Simulation Design**

In order to validate this error estimator we simulate measurement cases using different observations. The simulation design consists in defining the different positions of the observations associated with the  $(\beta_k)_{1 \le k \le m}$  orientations. We then add disturbances to the values  $(\delta O_k^{mes})_{1 \le k \le m}$  and the orientations  $(\delta \beta_k^{mes})_{1 \le k \le m}$  of the observations and we look at the evolution of the errors between the assumed values of the indicators and the identified values using the data fusion. This design aims at studying the robustness of the proposed data fusion process to the measurement errors, and to validate the error estimator. For each of these simulations we evaluate the indicator error  $\delta I_{\sigma}$ , the observation error  $\delta O_{\sigma}$ ,  $h_s$  and  $E_{hs}$ . The error of the indicators is computed by the  $L^2$ -norm of the distance of the reference solution to the identified solutions. The error of the observation as shown in figure 4 when one or two observations are disturbed. These both errors are normalized with respect to



Figure 4: Illustration of the errors of the observations and of the indicators when one and two observations are disturbed.

 $\sigma$  which is the same for all the observations. Lastly, the relative positions of the consecutive observations are chosen from 15° to 75° and the disturbances are included in the intervals  $\delta\beta \in [-10^\circ, 10^\circ]$  and  $\delta O^{mes} \in [-5\sigma, 5\sigma]$ .

#### Results

Figure 5 shows the evolution of  $\delta I_{\sigma}$  as a function of  $\delta O_{\sigma}$  for the whole simulated cases. 22% of these cases increase measurement errors because  $\delta I_{\sigma} > \delta O_{\sigma}$  and 78% of these cases decrease the measurement errors because  $\delta I_{\sigma} \leq \delta O_{\sigma}$ . This first result shows that the data fusion allows to efficiently reduce the measurement uncertainties. The second graph in figure 5 is a reduced version of the first one, only for two different positions of the observations. The first position (blue lines) is a  $[15^{\circ}, 30^{\circ}]$  relative position and the second position (red lines) is a  $[30^{\circ}, 75^{\circ}]$ . The number of each line corresponds to the 1 to 3 observation that is disturbed. This graph shows that the sensitivity to the measurement errors is linked to the relative position of the observations but also to the disturbed observation. Looking for the mean values of the  $\delta I_{\sigma} \delta O_{\sigma}$  evolution has to be associated with the maximum relative position because the sensitivity to the measurement errors quickly increases when the observation has to be associated with the maximum relative position because the sensitivity to the measurement errors quickly increases when the observation is disturbed. Lastly, the intermediate relative



Figure 5: Evolution of the normalized indicator error  $\delta I_{\sigma}$  as a function of the normalized observation error  $\delta O_{\sigma}$ .

position has to be associated with the worst reliable observation because it leads to the smallest change in the sensitivity to the measurement errors. These results define some simple criteria in order to obtain the appropriate selection of observations in a database.

Figure 6 shows the evolutions of  $\delta I_{\sigma}$  as a function of  $h_s$  and  $E_{hs}$ . These evolutions are classified depending on the increasing convergence rate of the observations. The value of the converge rate associated with the different colors depends on the position of the three observations' intersection. This rate is computed by:

$$\tau_c = \frac{1}{m} \sum_{k=1}^m \max\left[\min\left(\min_{1 \le j \ne k \le m} \left[\pi_j, \pi_k\right], \min_{1 \le i \ne k, j \le m} \left[\pi_j, \pi_k\right]\right)\right]$$
(5)

 $\tau_c = 1$  when the distributions of the three observations are equal to 1 in the same point.  $\tau_c = 0$  when at least one of the observations is equal to 0 which means that there is no point such that the three observations are simultaneously different from 0.



Figure 6: Evolution of the normalized indicator error  $\delta I_{\sigma}$  as a function of the threshold level  $h_s$  and the error estimator  $E_{hs}$ .

Although the  $h_s$  is simply related to the convergence of the observations, we note that for a same value of  $h_s$  in the same observation configuration,  $\delta I_{\sigma}$  can significantly increase, from 0 to 4 when  $h_s$  is minimized. The  $E_{hs}$  evolution is also related to the observation convergence, but the variation of  $\delta I_{\sigma}$  decrease when  $E_{hs} = 1$  for each configuration. This result confirms that  $E_{hs}$  quantifies the reliability of the solution and defines an intrinsic error estimator associated with the convergence of the observations. The  $\varepsilon$  value is then adapted depending on the choice of the observations in the database. For example, using the  $[60^\circ, 60^\circ]$  relative positions of the three observations with the same reliability  $\sigma$  we obtain an optimum  $\varepsilon$  value of 0.5. Lastly, similar results are obtained considering from four to six observations and simultaneous disturbances.

### **CONCLUSION and PROSPECTS**

In this paper, we studied the reliability of the data fusion method we proposed for the identification of the concrete material properties. This method is based on the use of multiple NDT techniques in order to reduce the uncertainties of the identified properties. The variability of each NDT technique is integrated in the data fusion method such as their reliability in experimental procedures or their accuracy to detect the material evolutions. We proposed a numerical testing of this method based on the relative position of the observations in the space of the indicators. This testing allowed to define a methodology for the choice of the optimum observations depending on their sensitivity to the measurement errors. The relative positions of the observation are maximized and the worth reliable observations are located at the intermediaire positions. Lastly, an intrinsic error estimator was defined in order to estimate the accuracy of the identified solution. This error estimator was associated with the convergence of the observation and the weight of the identified solution compared to the observation variabilities. The numerical testing then showed that this error estimator achieves the objective of estimating the accuracy of the identified solution.

Future work will concern the application of this methodology to a real NDT database. The optimum choice of the observations will be first tested and the error estimator will be validated. The final objective of this work is to propose a global tool that accurately estimates the material properties of concrete structures in situ.

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