Application of Compressed Sensing to 2-D Ultrasonic Propagation Imaging System Data

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ABSTRACT

The Ultrasonic Propagation Imaging (UPI) System is a unique, non-contact, laser-based ultrasonic excitation and measurement system developed for structural health monitoring applications. The UPI system imparts laser-induced ultrasonic excitations at user –defined locations on a structure of interest. The response of these excitations is then measured by piezoelectric transducers. By using appropriate data reconstruction techniques, a time-evolving image of the response can be generated. A representative measurement of a plate might contain 800x800 spatial data measurement locations and each measurement location might be sampled at 500 instances in time. The result is a total of 640,000 measurement locations and 320,000,000 unique measurements. This is clearly a very large set of data to collect, store in memory and process. The value of these ultrasonic response images for structural health monitoring applications makes tackling these challenges worthwhile.

Recently compressed sensing has presented itself as a candidate solution for directly collecting relevant information from sparse, high-dimensional measurements. The main idea behind compressed sensing is that by directly collecting a relatively small number of coefficients it is possible to reconstruct the original measurement. The coefficients are obtained from linear combinations of (what would have been the original direct) measurements. Often compressed sensing research is simulated by generating compressed coefficients from conventionally collected measurements. The simulation approach is necessary because the direct collection of compressed coefficients often requires compressed sensing analog front-ends that are currently not commercially available. The ability of the UPI system to make measurements at user-defined locations presents a unique capability on which compressed measurement

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techniques may be directly applied. The application of compressed sensing techniques on this data holds the potential to reduce the number of required measurement locations, reduce the time to make measurements, reduce the memory required to store the measurements, and possibly reduce the computational burden to classify the measurements. This work considers the appropriate selection of the signal dictionary used for signal reconstruction, and performs an evaluation of compressed sensing technique's ability to reconstruct ultrasonic images using fewer measurements than would be needed using traditional Nyquist-limited data collection techniques.

INTRODUCTION

Compressed sensing has been a prolific research topic in applied math and statistics over the last few years. Excellent tutorials covering the basics of compressed sensing can be found in [1], [2], and [3]. To summarize, a signal of interest x can be represented as:

$$x = \sum_{i=1}^{N} s_i \psi_i \quad \text{or in matrix form as } x = \Psi s \tag{1}$$

Where Ψ is an orthonormal basis and "s" is the representation of the signal in the Ψ domain. In the case of compressed sensing we are interested in the case where x is compressible in some domain. That is, the number of significant non-zero elements of s is equal to K and $K << N$. K is known as the "sparsity" of the signal. A measurement matrix "Φ" is then introduced to produce compressed sensing coefficients y.

$$y = \Phi x = \Phi \Psi s = \Theta s \tag{2}$$

Where Φ has $M<<N$ rows. At this point it is important to note that this equation represents an underdetermined system of linear equations. One of the major breakthroughs of the compressed sensing community was the finding that assuming $K<<N$ it is possible to recover x from y assuming the matrix Φ possesses the restricted isometry property (RIP) and x is sparse in some basis [5]. An example of a sparse signal would be a signal that only contains a few non-zero Fourier coefficients such as a sum of decreasing harmonics that represents a structure's impulse response. The direct formulation of this problem is finding the vector s with minimal ℓ_0 norm. Unfortunately ℓ_0 norm minimization is numerically unstable and computationally expensive [2]. It has been shown though that the ℓ_0 can be replaced with an ℓ_1 norm relaxation [1]. The ℓ_1 norm regularization problem [2], [4] can be solved to recover sparse signals from the compressed coefficients y. Amazingly, it is possible to recover x using an M measurements following the relation [2]:

$$M \geq cK \log\left(\frac{N}{K}\right) \tag{3}$$

Where c is a constant that has empirically been found to approximately equal 4.0 [2]. Equation (3) implies that it is possible to reconstruct a sparse signal x using far fewer measurements than elements in x. In this work, the ℓ_1 norm regularization approach will be explored for recovering the signal x from compressed coefficients. The ℓ_1 norm regularization problem used to attempt recovery of the original signal can be written as:

$$\underset{s}{Minimize}\left(\left\| y - \Phi \Psi s \right\|_2 + \gamma \left\| s \right\|_1\right) \tag{4}$$

Here "s" is the vector of coefficients for the dictionary vectors in Ψ. The parameter γ is used to trade-off between the sparsity in the coefficients of s and the quality of the fit to the compressed measurements y. In the extreme case where γ is set equal to zero the problem degenerates into the conventional ℓ_2 norm problem. Prior work done by the authors has shown a range of γ values produce similar compressed sensing results [5]. For this work γ was set equal to 0.1.

ULTRASONIC PROPOGATION IMAGING SYSTEM COMPRESSED SENSING CONCEPT

The conventional UPI system was introduced in [6]. To sumarize, a Q-switched laser is used to excite a structure at ultrasonic frequencies by directing thermal energy into the structure. The response of the structure is then measured using a PZT sensor. The laser is used to provide multiple excitations by performing a raster-scan sweep across the structure of interest. The individual time responses can then be arranged in a pattern corresponding to the spatial arrangement of the raster scan. The result is generally a 3-D cube of data. Two of the dimensions correspond to the spatial dimensions, and one of the dimensions corresponds to time. It is possible to generate 2-D snapshots of the data at each instant of time. By sequentially stepping through these snapshots or time-slices of data it is possible to generate a movie that illustrates the ultrasonic wavefield propogating through the structure as time evolves. Damage in the structure of interest manifests itself as anomalies in the wavefield propogation. The UPI measurement technique features a number of advantages. It can be used to collect high-resolution ($<$ 1 mm spatial resolution) full-field ultrasonic data using only a single embedded PZT sensor. It is possible to literally watch a movie of ultrasonic waves propogating in thin plate structures. One challenge associated with the UPI system is that in its raw form a relatively large amount of data is typically generated from these measurements. A typical data cube from the UPI might be on the order of 560x252x500 ≈70.6 million data points which is a fairly high dimensional measurement when compared to a typical time series for SHM. At first glance though it is obvious that the data collected by this system is not unlike data collected by a conventional camera in the sense that it has similar dimensionality. Techniques from the image processing community can be brought to bear on this challenge.

Compressed sensing is a novel sampling technique for collecting high-dimensional data such as images. The high-dimensional nature of UPI data, coupled with the method used to collect the measurements makes UPI a good candidate for compressed sensing techniques. *Figure **1*** summarizes the proposed UPI compressed sensing concept. The following summarizes how a compressed measurement is collected. First, a Q-switched excitation laser is directed onto a digital micro mirror array. The digital micro mirror array implements the incoherent Φ matrix used to perform the compressed measurement process. It does this by directing an incoherent pattern of laser beams onto the structure of interest. The resulting response is then measured using a single PZT sensor as normal. This response will be a linear superposition of the responses that would have been collected from each laser beam individually. This processor corresponds to generating a single compressed measurement/element of the y vector in equation 2. The process is then repeated using a different incoherent pattern of excitation laser beam until a sufficient number or compressed measurements is collected.

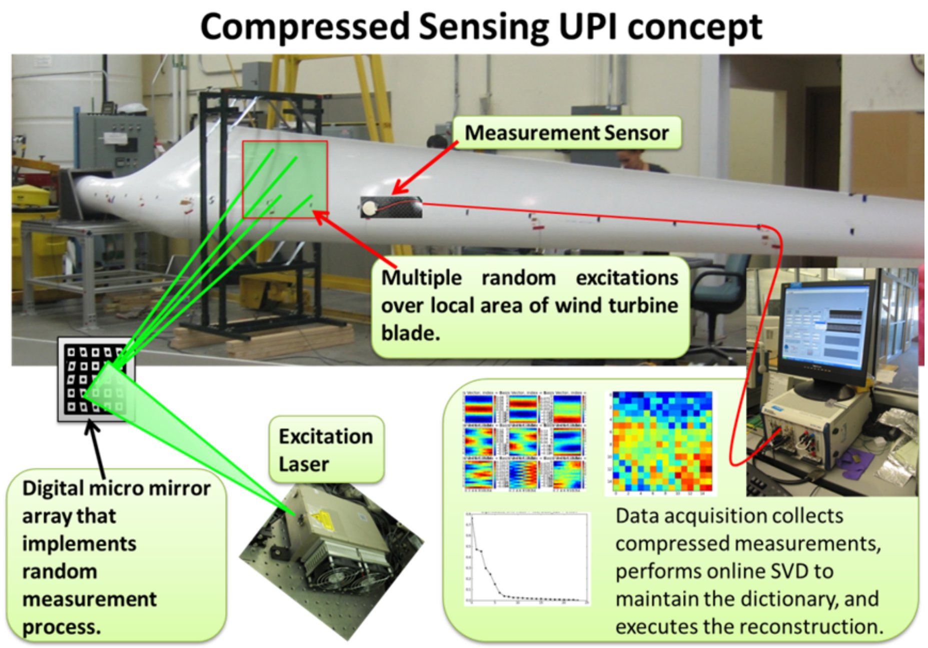

Figure 1. The proposed UPI compressed sensing measurement process.

This concept is not unlike the single pixel camera described in [7]. Once the compressed measurements have been collected, the data is sent to a central server. The server would perform a variety of functions. For example, the server would maintain the dictionary Ψ and could potentially perform online singular value decomposition (SVD)-based principal components analysis (PCA) to update the dictionary [8], [9] as new expressions of damage are uncovered by the system. The server would also be used to perform compressed sensing reconstruction, and smashed filtering as was described in [5], [10].

The goal of this initial research effort is to investigate the feasibility of using compressed sensing techniques for UPI data collection. The hope is that by using these techniques it will be possible to collect substantially fewer measurements leading to increases in scan rates and a decrease in the memory requirements for storing this data. Furthermore, the high dimensionality of this data implies that data processing may become quite cumbersome. It is highly possible that by making use of compressed sensing techniques such as the smashed filter that the time and memory requirements for SHM analysis will be significantly reduced.

GENERATION OF MEASUREMENT MATRIX Φ

Often the compressed sensing literature will discuss the use of a measurement matrix Φ whose elements are derived from values sampled i.i.d from a Gaussian random distribution [11]. Unfortunately Gaussian random matrices will not work for the proposed compressed sensing concept for two reasons. The first reason is that the digital micro mirror array used to implement the compressed sensing inherently is only able to assume binary on or off states. It is not able to assume the continuous-valued, positive and negative numbers required by the Gaussian distribution. The second concern is that the laser is only able to provide a finite amount of power into the plate. The more beams that are used to excite the plate, the smaller the amount of energy is available to excite the plate. If a very large number of beams are used, each individual beam may have so little energy associate with it that the response is not detectable with the PZT sensor. From a laser energy standpoint it is preferable to excite the plate using as few beams as possible. A Gaussian Φ would require that some laser energy be imparted to every dimension in the conventional measurement. Furthermore, it would need both positive and negative excitation which is not possible with the current UPI system. In order to address this problem, these simulations have begun to investigate the possibility of using the sparse, binary matrices discussed in [12]. These matrices do not have the same robustness and performance with respect to compressed sensing reconstruction that Gaussian matrices enjoy. Regardless their sparse, binary nature make them work investigation for the UPI application. Sparse binary matrices were used throughout this work. The matrices were generated by selecting the number of non-zero elements desired in each column of the Φ matrix. A binary column vector with the appropriate number of ones was then generated. The elements of the column vector were then randomly shuffled. This procedure was then repeated for each dimension of the conventional measurement and the resulting column vectors were concatenated together to form a measurement matrix Φ.

CHOICE OF DICTIONARY FOR UPI MEASUREMENTS

When executing compressed sensing reconstruction it is imperative that a suitable dictionary of vectors Ψ be used to perform the reconstruction. It generally has been found helpful to use a dictionary that includes vectors that closely match the measurement that is being reconstructed [10]. *Figure 2* shows a sequential progression some of the measurements collected during the course of this work. These measurements are taken from a pipe specimen that has an ellipsoidal-shaped reduction in thickness in the center of the measurement. The specimen is reduced to 40% of its nominal thickness. When the wave front meets the reduction in thickness at time slice 300 it is possible to observe the reflections that result from the stiffness change.

The measurements displayed in *Figure 2* suggest that a Fourier basis may be a good choice for signal reconstruction. Other possible choices include wavelets and Gabor atoms. In this work the possibility of using dictionaries learned from data is also explored. Specifically, a learned dictionary is generated by taking a number of conventional measurements and converting them into column vectors. The column vectors can then be assembled into a data matrix. Principal components analysis can then be applied to this matrix. The eigenvectors associated with the large eigenvalues can then be used to augment the dictionary. The idea is that the eigenvalues should naturally be a good fit to the data since they were generated from similar measurements. This idea was explored by taking the 400th time slice and breaking it into 16 x 16 patches. These patches were then converted into column vectors with 256 elements and assembled into a data matrix. Principal components analysis was then performed on the covariance matrix of the data matrix by using singular value decomposition. The resulting eigenvector of the covariance matrix can be seen in *Figure 3*. PCA analysis shows that the rank of the data matrix is very small in

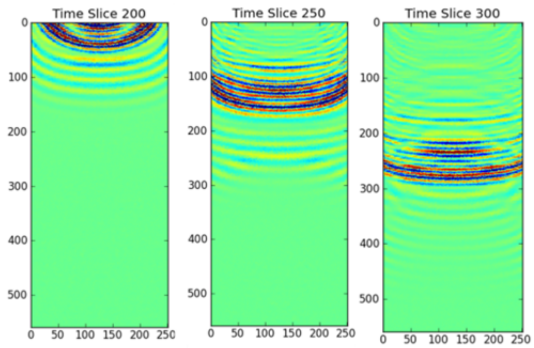

Figure 2. Examples of measurements collected by the UPI system. Color bar indicates scaled units. Measurements were obtained with a Fuji R-Cast acoustic emission sensor.

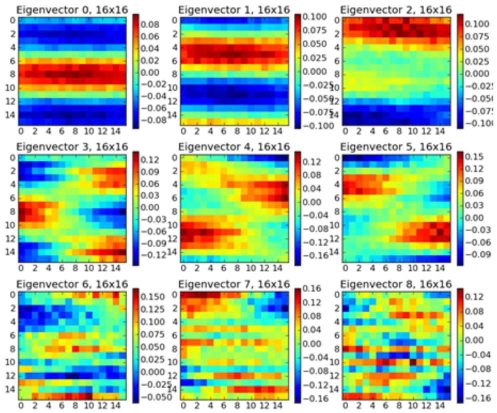

Figure 3. The first 9 eigenvectors generated from the covariance matrix of time slice 400 when divided into 16x16 patches.

comparison to its dimensionality. The data has a dimensionality of 256, but the majority of the variance in the data is accounted for with only the first 10 eigenvectors. These results suggest that it may be possible to obtain suitable reconstructions of these types of measurements using under-complete dictionaries learned from the data.

RECONSTRUCTION RESULTS

A preliminary check was performed to do an initial feasibility study of compressed sensing. The 410th time slice from the measurement described in the previous section was broken up into 16x16 blocks. The 40th block was selected for demonstrating simulated compressed sensing (*Figure* ***4***(a)). The number of compressed coefficients collected was $M=128$. An over-complete dictionary was created by concatenating the dictionary generated from the PCA analysis of the 400th time slice with a 16x16 2-Discrete Fourier Basis. Only the eigenvectors corresponding to the 50 highest eigenvalues were used from the PCA-generated dictionary. The γ parameter was set equal to 0.1. Reconstruction was executed using the optimization routine described in equation (4) and was implemented with the CVXMOD software [13]. The result of the reconstruction is shown in *Figure* ***4***. *Figure* ***4***(a) shows the original data block, and *Figure* ***4***(b) shows the reconstructed data block. The reconstruction error was calculated using normalized Root Mean Square Deviation (RMSD) expressed as a percentage:

$$\overleftarrow{error} = \overleftarrow{reconstruction} - \overleftarrow{measurement} \tag{5}$$

$$RMSD_{normalized} = \frac{1}{\max(\overleftarrow{error}) - \min(\overleftarrow{error})} * \sqrt{\frac{\sum_{i=1}^{N}(\overleftarrow{error}_i)^2}{N}} * 100\% \tag{6}$$

Where $\overleftarrow{reconstruction}$ is the reconstructed data, $\overleftarrow{measurement}$ is the original data, and N=256 is the dimensionality of the original data. The percentage normalized RMSD for this reconstruction is 15.65%. Depending on the application this may or may not be sufficient. It is important to not however that only half as much data was collected, and the time to collect the measurements would be cut in half if a dedicated compressed sensing front-end were employed.

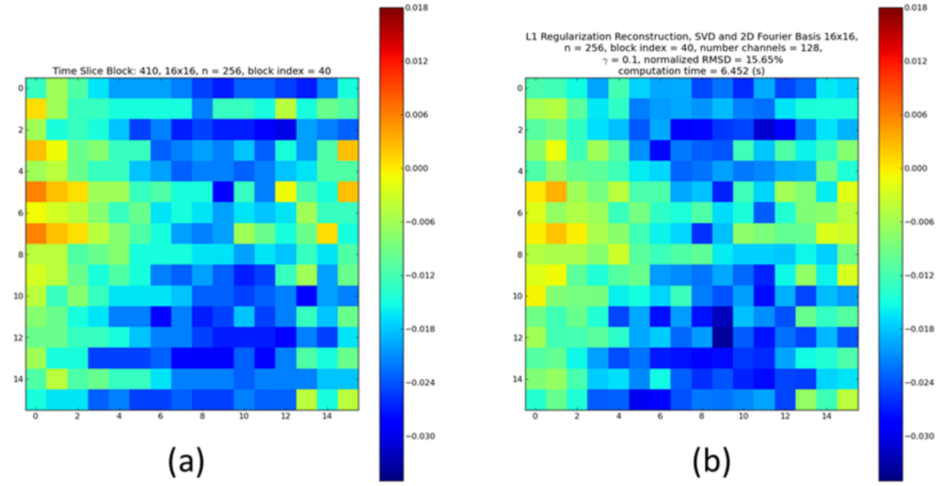

Figure 4. (a) Original 16x16 block of data, (b) Reconstructed 16x16 block of data.

CONCLUSIONS

These results obtained in this work are promising enough to continue investing effort into exploring compressed sensing for the UPI application. It is expected that greater performance gains will be realized if the size of the reconstructed blocks can be increased. Computational limitations are currently preventing the timely reconstruction of blocks larger than 32x32. It is expected that by making use of greedy algorithms such as CoSaMP [14] for the reconstruction as well as Graphical Processor Unit (GPU) technology, it will become practical to reconstruct increasingly large data blocks. Effort will also continue in making use of deep learning techniques [15] to learn appropriate dictionaries for reconstruction from the data.

The initial results presented in this work suggest that compressed sensing shows promise as a candidate technology to speed up the collection of UPI data, as well as help ease the memory requirements to store UPI data. Work will continue to develop more high-performance UPI compressed sensing systems.

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