Sequential Structural Health Monitoring and Damage Detection

J. KULLAA

ABSTRACT

Structural health monitoring (SHM) and damage detection performed in time domain from the measured vibration data is studied. The advantages of damage detection in time domain include: 1) direct sensor data can be used with no complex feature extraction and 2) the data dimensionality remains low. The main disadvantage is that the amount of data easily becomes exhaustive. Therefore, the covariance matrix estimation can be difficult. The present paper discusses sequential on-line SHM, in which damage detection is performed each time when a new measurement is available. Assuming the analysis parameters remain the same, many functions can utilize recursive (sequential) estimation to save time and memory. Only the projection to the principal subspace must be repeated for all data, because the principal subspace may vary. An experimental study is performed to validate the proposed algorithm.

INTRODUCTION

In vibration-based structural health monitoring (SHM), damage identification is performed from time histories measured simultaneously with several accelerometers or strain gauges at different locations of the structure. Damage detection can be performed in the time domain from the raw sensor data or in the feature domain, in which damage-sensitive features are first extracted from the time series. Both alternatives have their advantages and disadvantages. The advantages of damage detection in time domain used in this study include the following: 1) direct sensor data can be used with no complex feature extraction, and 2) the data dimensionality remains low. The main disadvantage is that the amount of data may be exhaustive.

Jyrki Kullaa, Department of Applied Mechanics, Aalto University, P.O. Box 14300, 00076 Aalto, Finland. Alternative address: Helsinki Metropolia University of Applied Sciences, P.O. Box 4021, 00079 Metropolia, Finland (jyrki.kullaa@metropolia.fi).

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A single vibration measurement typically consists of hundreds of samples from each sensor. The model of the undamaged structure is build using training data acquired under different environmental or operational conditions during a long time. Hence, in a time-domain approach, the amount of training data easily becomes excessive.

Many damage detection algorithms utilize the data covariance matrix. Due to the vast amount of training data, the covariance matrix estimation can be difficult. The monitoring period is typically several years resulting also in a huge amount of test data.

In the on-line SHM, the decision about damage existence is made after each new measurement. If the model parameters remain the same, the sensor network model must be built only once. This model is then fixed before launching the SHM system. It will be seen that the features for the new test data can be created using the fixed model and the new data only. Many functions can utilize recursive (sequential) estimation. However, the features from all previous measurements are needed for dimensionality reduction. Also the control charts for damage detection must be re-designed.

The paper is organized as follows. First, the generalized likelihood ratio test (GLRT) for damage detection is described. The damage detection algorithm is reported utilizing sequential estimation. An experimental study is performed to validate the proposed sequential analysis algorithm for on-line damage detection and localization. A short conclusion is given in the end.

FEATURES FOR DAMAGE DETECTION AND LOCALIZATION

The sensor network is modelled as a Gaussian process [1]. This process can be both spatial and temporal. In mechanical vibrations, spatial and temporal correlations are related to the mode shapes and natural frequencies, respectively. The Gaussian process model is fully determined by its mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$:

$$p(\mathbf{x}) = |2\pi\boldsymbol{\Sigma}|^{-1/2} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right] \quad (1)$$

where $p(\mathbf{x})$ is the probability density function (pdf) and $\mathbf{x}$ is the measured variable, typically a simultaneous sample of accelerations or strains.

In principle, the model parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ can be easily estimated directly from the measurements. However, two issues emerge. First, the amount of training data is often exhaustive, consisting of several measurements under different environmental or operational conditions during a long time. Therefore, the estimation of the model parameters must be performed recursively. Second, due to the curse of dimensionality, the number of model parameters is high, and it would be necessary to perform dimensionality reduction. This is done by computing the likelihood ratio separately for each sensor in the network and performing principal component analysis (PCA) in the end.

To estimate a model for a single sensor, the minimum mean square error (MMSE) estimation is used, in which the signal of each sensor is estimated in turn using the other sensors in the network. It is assumed that the number of sensors is adequate to build up a redundant system; for spatial correlation, this number should be greater than the number of active modes. Spatiotemporal correlation can be utilized if the process can be assumed stationary [2]. The sensors are divided into observed sensors **v** and missing sensors **u**:

$$\mathbf{x} = \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} \tag{2}$$

with a partitioned covariance matrix $\mathbf{\Sigma}$ of the training data

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{uu} & \mathbf{\Sigma}_{uv} \\ \mathbf{\Sigma}_{vu} & \mathbf{\Sigma}_{vv} \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}_{uu} & \mathbf{\Gamma}_{uv} \\ \mathbf{\Gamma}_{vu} & \mathbf{\Gamma}_{vv} \end{bmatrix}^{-1} \tag{3}$$

where the precision matrix $\mathbf{\Gamma}$ is defined as the inverse of the covariance matrix $\mathbf{\Sigma}$ and is also written in the partitioned form. A linear MMSE estimate is [2]:

$$\hat{\mathbf{u}} = \boldsymbol{\mu}_u - \mathbf{\Gamma}_{uu}^{-1}\mathbf{\Gamma}_{uv}(\mathbf{v} - \boldsymbol{\mu}_v) \tag{4}$$

where $\boldsymbol{\mu}_u$ and $\boldsymbol{\mu}_v$ are the mean of **u** and **v**, respectively. The error covariance matrix is

$$\mathbf{\Phi} = \mathrm{cov}(\mathbf{u}|\mathbf{v}) = \mathbf{\Gamma}_{uu}^{-1} \tag{5}$$

MMSE results in a conditional distribution for each sensor in the network. Assuming Gaussian distribution, the conditional pdf of **u** becomes:

$$p(\mathbf{u}|\mathbf{v}) = |2\pi\mathbf{\Phi}|^{-1/2} \exp\left[-\frac{1}{2}(\mathbf{u} - \hat{\mathbf{u}})^T \mathbf{\Phi}^{-1}(\mathbf{u} - \hat{\mathbf{u}})\right] \tag{6}$$

Damage detection is done using the hypothesis test for the MMSE model parameters [2] applying the generalized likelihood ratio test (GLRT) [3]. The test statistic is the log-likelihood ratio $l(\mathbf{u}|\mathbf{v})$ for each sample:

$$l(\mathbf{u}|\mathbf{v}) = \ln \frac{p(\mathbf{u}|\mathbf{v}; H_1)}{p(\mathbf{u}|\mathbf{v}; H_0)} \tag{7}$$

where $p(\mathbf{u}|\mathbf{v}; H_i)$ is the probability according to the hypothesis H_i, i = 0, 1. The hypothesis H_0 is that the model parameters are the same as those of the training data (normal), and the hypothesis H_1 is that the parameters are different to those of the training data (anomaly). The distributions $p(\mathbf{u}|\mathbf{v}; H_0)$ and $p(\mathbf{u}|\mathbf{v}; H_1)$ are obtained by estimating the parameters from the training data and the current measurement, respectively. The hypothesis test is based on the Neyman-Pearson (NP) lemma [3]. To maximize the probability of detection P_D for a given probability of false alarm $P_{FA} = \alpha$, decide H_1 if $\sum l(\mathbf{u}|\mathbf{v}) > \gamma$. The threshold γ is found from the false alarm constraint $P_{FA} = \alpha$. Choosing the threshold for detection is discussed in the following.

Each sensor yields one test statistic, resulting in a total number of variables equal to the number of sensors in the network. The dimensionality can be reduced by using principal component analysis (PCA) [4] to the log-likelihood ratios l (Equation 7). The distributions of l and the PCA scores are unknown. Therefore, the statistics used for novelty detection are the maxima and minima of the first principal component scores, and using the theory of extreme value statistics (EVS) [5, 6], the thresholds are designed so that the probability of false alarms is 0.001. In this study, the extreme values are computed from 500 subsequent variables. Finally, the statistics are plotted on a control chart [7].

Damage localization is performed by computing the average log-likelihood ratio for each sensor in the measurement. The largest value is assumed to reveal the sensor closest to the damage location.

RECURSIVE ESTIMATION (TRAINING DATA)

The estimation of the mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ from a high amount of training data needs special consideration. Here, a recursive method for estimation is presented. It is assumed that the data are divided into segments, each representing a multidimensional time series measurement.

The estimates of the mean and the matrix of mean sums of squares and products for measurement j are respectively:

$$\begin{aligned} \bar{\mathbf{x}}_j &= \frac{1}{n_j}\sum_{k=1}^{n_j}\mathbf{x}_k \\ \overline{\mathbf{M}}_j &= \frac{1}{n_j}\sum_{k=1}^{n_j}\mathbf{x}_k\mathbf{x}_k^T \end{aligned} \tag{8}$$

where n_j is the number of samples in measurement j. Also temporal correlation can be estimated by time-shifting of the data [2].

If the current (i) estimates of the mean and the matrix of mean sums of squares and products are $\hat{\boldsymbol{\mu}}_i$ and $\hat{\mathbf{M}}_i$, respectively, and those of the subsequent measurement j are $\bar{\mathbf{x}}_j$ and $\overline{\mathbf{M}}_j$, then the updated (i+1) estimates become:

$$\begin{aligned} \hat{\boldsymbol{\mu}}_{i+1} &= \hat{\boldsymbol{\mu}}_i + n_j P_{i+1}(\bar{\mathbf{x}}_j - \hat{\boldsymbol{\mu}}_i) \\ \hat{\mathbf{M}}_{i+1} &= \hat{\mathbf{M}}_i + n_j P_{i+1}\left[\overline{\mathbf{M}}_j - \hat{\mathbf{M}}_i\right] \end{aligned} \tag{9}$$

where

$$P_{i+1} = \frac{1}{P_i^{-1} + n_j} \tag{10}$$

Initially:

$$\hat{\boldsymbol{\mu}}_0 = \mathbf{0}, \quad \hat{\mathbf{M}}_0 = \mathbf{0}, \quad P_0^{-1} = 0 \tag{11}$$

Finally, after recursive estimation, the covariance matrix estimate is:

$$\hat{\boldsymbol{\Sigma}} = \hat{\mathbf{M}} - \hat{\boldsymbol{\mu}}\hat{\boldsymbol{\mu}}^T \tag{12}$$

Often, standardized variables are preferred with a zero mean and unit standard deviation. The standardized variable **z** is:

$$\mathbf{z} = \mathbf{S}^{-1}(\mathbf{x} - \hat{\boldsymbol{\mu}}) \tag{13}$$

where the diagonal standard deviation matrix **S** is:

$$\mathbf{S} = \left[\mathrm{diag}(\hat{\boldsymbol{\Sigma}})\right]^{\frac{1}{2}} \tag{14}$$

Transformation (13) results in the following covariance matrix for the standardized variables:

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{z}} = \mathbf{S}^{-1}\hat{\boldsymbol{\Sigma}}\mathbf{S}^{-1} \tag{15}$$

The algorithm to build a model of the training data is as follows.

1. Monitoring the undamaged structure at different environmental or operational conditions. Data acquisition and signal processing of the training data.
2. Define the model parameters.
3. Estimate the mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ of the training data using a recursive algorithm.
4. Extract features (log-likelihood ratios) for each measurement.
5. Compute the mean and covariance matrix of the features for PCA using a recursive algorithm.

DAMAGE DETECTION AND LOCALIZATION (NEW TEST DATA)

Once new data arrive, the objective is to decide if damage is present or not. It is assumed that the mean and covariance of the training data have already been estimated and fixed. Also the log-likelihood ratios have been computed for the training data. In the following on-line monitoring algorithm, the decision is made after each new measurement.

1. Acquire new measurement.
2. Extract features (step 4 in the previous list).
3. Update the mean and covariance matrix of the features (estimated in step 5 of the previous list) for PCA using a recursive algorithm.
4. PCA: Compute the first principal component of the updated covariance matrix.
5. Projection of the features of all previous measurements onto the first principal component.
6. Damage detection using control charts.

7. Damage localization.
8. Return to step 1.

In steps 4–6, recursive estimation is not utilized. For PCA, recursive algorithms exist [8], but the main issue is the projection of the features of all previous measurements onto the first principal axis. The projection must be repeated after each new measurement, because the principal subspace may vary.

Notice that two different covariance matrices are needed. Both matrices may need a recursive algorithm. Let the number of sensors be p and the model order m.

1. The covariance matrix of the measurement data to build the sensor network model for feature extraction (Equation 1). This matrix includes information of the training data only. The dimensionality is $p(m+1)$.
2. The covariance matrix of the feature vector for PCA. This matrix includes information of all previous measurements. The dimensionality is p.

EXPERIMENTAL RESEARCH

The proposed approach was investigated with a monitoring system built in the laboratory. The structure was a 4.2 meters long, 36 kg wooden model bridge shown in Figure 1. Random excitation was applied to the structure to excite the lowest modes. Fifteen accelerometers measured the response at three different longitudinal positions. The sampling frequency was 256 Hz and the measurement period was 32 s. For sufficient redundancy, the data were low-pass filtered with 128 Hz and re-sampled resulting in 4076 samples per channel in each measurement.

The measurements were made during several days, and it was noticed that the dynamic properties of the structure varied due to environmental changes. Temperature and humidity variations were assumed to be the main influences on the wooden structure.

Damage was then introduced by adding point masses on the structure. The sizes of the masses were 23.5, 47.0, 70.5, 123.2, and 193.7 g. The point masses were attached on the top flange, 600 mm left from the midspan (Figure 1). The last measurements were again from a healthy structure. The added mass was very small compared to the total weight of the structure (36 kg), even the highest mass increase was only half a percent. See Table 1 for details.

Model order 5 was used in the analysis. The training data were measurements 1–1860. The same measurements were used as the in-control data in the control chart design. The test data were the remaining measurements 1861–2010. Extreme value statistics were used in the control charts with a subgroup size of 500 resulting in eight plotted statistics for each measurement.

On-line damage detection was performed by applying PCA after each new measurement. The 15 log-likelihood ratios were projected onto the first principal component and a control chart was designed for the projected data. Therefore, the control chart had to be re-designed each time after new data arrived resulting also in a change of the control limits. The statistics and the corresponding control limits of the latest measurement were only plotted and no information about the future measurements was available.

Figure 2 left shows the on-line EVS control chart for the test data. The two solid lines represent the minima and maxima in each subgroup and the dashed lines are the corresponding control limits. It can be seen that the detection performance is excellent with no false positives or false negatives. It should be emphasized that since this was an on-line analysis, the figure also shows that each damage level was detected immediately. The damage size is also visible.

For damage localization, the mean log-likelihood ratio of each sensor is plotted in Figure 2 right. Sensor 7 shows the highest value. From Figure 1 it can be seen that damage was located halfway between sensors 7 and 8. Sensor 8 also showed a high number. However, sensors 1 or 2 measuring vertical acceleration did not indicate damage. The damage size is also visible.

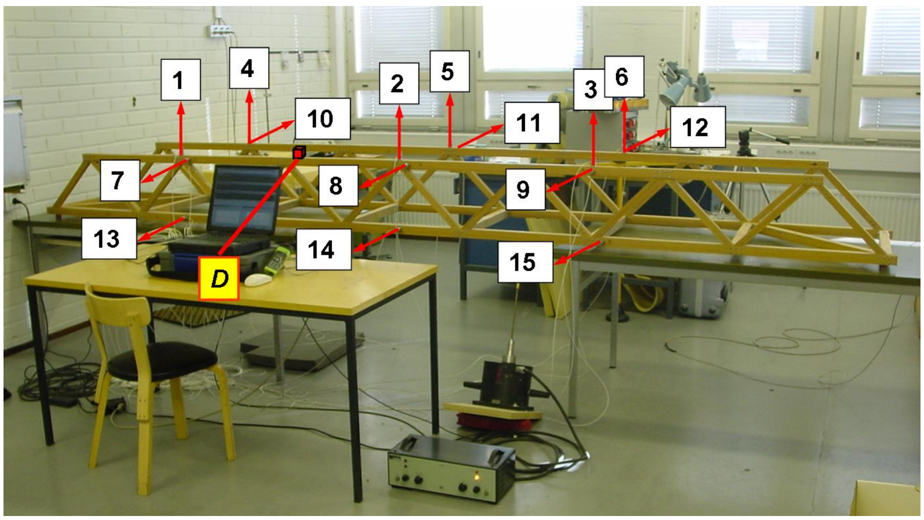

Figure 1. Wooden bridge test setup. Accelerometer and damage (D) locations.

Table 1. Increase of the structure's mass at each damage case and the corresponding measurement numbers.

Damage Case	Added Mass (g)	Mass Increase (%)	Measurement Numbers	Number of Measurements
U	0	0	1–1882	1882
D_1	23.5	0.065	1883–1902	20
D_2	47.0	0.13	1903–1925	23
D_3	70.5	0.20	1926–1947	22
D_4	123.2	0.34	1948–1967	20
D_5	193.7	0.54	1968–1987	20
U	0	0	1988–2010	23

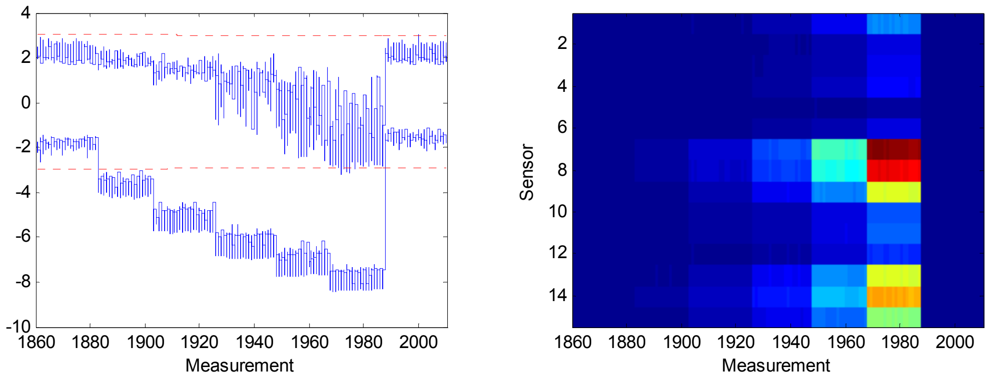

Figure 2. Left: EVS control chart for damage detection. Right: Damage localization.

CONCLUSION

Vibration-based on-line structural health monitoring (SHM) in time domain is studied. Recursive estimation of the model parameters is often needed. After launching the SHM system, the feature covariance matrix can be updated recursively, whereas PCA and the projection of all data to the principal subspace must be repeated. For PCA, a recursive algorithm [8] was tested, but the result was not satisfactory. Nevertheless, PCA for a covariance matrix of size 15×15 was not an issue. Instead, the projection of the features of all previous measurements to the first principal axis proved to be time consuming, because the data were stored in separate files.

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