Damage Identification Using Sub-Structuring and Optimal Modal Reduction Techniques

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ABSTRACT

The purpose of damage identification procedures in large structures is to assess the stiffness distribution in a specific zone (master), while minimizing the number of measurements on the other zones of the structure (slaves). In order to achieve this goal a sub-structuring strategy is usually adopted. The reduction in the measurement and the computational efforts is achieved by replacing the slave substructures with other ones with a much smaller number of sensors and Degrees of Freedom (DoFs), respectively. Since the reliability of the identified damage, involved in such condensation, is strongly dependent on the sensors location in the slave substructures, this study offers to use the Optimal Modal Reduction (OMR) technique

The OMR technique minimizes the error of the modal parameters (frequencies and mode shapes) of the master structure, in such a way that the DoFs obtained from this technique indicate the optimum sensors location in the slave sub-structures. The identification procedure is then applied only to the unknown parameters of the master structure.

This study demonstrates the efficiency of the OMR in damage identification procedure through multi-story shear building model. A Genetic Algorithm (GA), based optimization procedure, is applied for minimizing the differences between the simulated measured modal dynamic properties and the analytical one. In order to simulate field conditions the effect of noisy signals and limited number of sensors are considered.

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INTRODUCTION

For vibration-based structural health monitoring and damage identification procedures, it is not practical to assess a large structure with a complete set of measurements, mainly due to the limited number of sensors and the difficulty in field instrumentation. Moreover, identifying a large number of unknown parameters in a full structure causes difficulties in convergence of the numerical model. In order to handle this problem, a sub-structuring strategy is usually adopted. The main idea is to separate the structure into several substructures that are divided into master and slave substructures. The reduction in computational effort is achieved by replacing the slave substructures with equivalent ones with a much smaller number of Degrees of Freedom (DoFs).

Applications of model reduction techniques for damage identification procedures were mainly focused on the coupling forces between the substructures. When a large structure is divided into substructures, each substructure can be taken as an independent structural system. The coupling forces become pseudo dynamic loads acting on the adjacent substructures, and they can be identified from the measured responses of the substructures. The changes in the coupling forces can subsequently be used to identify local structural damage if there is any. Law et al. [1] offered to use measurement of one substructure to detect local damage in another substructure, by comparing the changes of the coupling forces acting between the substructures. The Virtual Distortion Method (VDM) for damage identification of trusses and beams, using the time domain approach, has been offered by Kolakowski et al. [2]. The method was improved by Swiercz et al. [3] for the frequency domain approach. Based on the VDM, Hou et al. [4] proposed a substructure isolation method. In their method the concerned substructure was numerically separated from the global structure by adding virtual supports on the substructure interface. In order to avoid the need for complete measurement, Tee et al. [5] offered to identify the entire structure using a divide-and-conquer approach that invokes for each substructure the concept of model condensation.

The main conclusion, which can be drawn from the above studies, is that the reliability of the identified damage is strongly dependent on the sensors location in the slave substructures. Moreover, the sensitivity of the measured response (strains, displacements, accelerations etc.) decreases when the distance between the local damage and the measured locations increases.

In order to avoid these problems, this study uses the Optimal Modal Reduction (OMR) technique, Givoli et al. [6], as a sub-structuring strategy. The OMR technique minimizes the error of the main structure response involved in such condensation. Therefore, the DoFs, in the slave sub-structures, obtained from the OMR technique indicate the optimum sensors location in the slave sub-structures. The identification procedure can then be applied only to the unknown parameters of the master sub-structure while assuming that the slave substructures are healthy ones.

The current study demonstrates the efficiency of the OMR in damage identification procedure. The identification procedure estimates a Virtual Damage Vector (VDV) by minimizing the differences between the modal parameters of the structural model and the measured one. The VDV is defined as the relative equivalent stiffness of each segment in the main structure. Thus the identification procedure searches the optimal VDV, in this study by using a Genetic Algorithm (GA) procedure.

Several studies used the GA for damage identification procedure, among them Mares et al. [7] which applied the GA, including rank-based selection, in truss structure. Friswell et al. [8] proposed the GA in cantilever beam structure, using an objective function that combined natural frequencies and mode shapes, and Ananda Rao et al. [9] that proposed a method for locating and quantifying damage in structural members by minimizing the differences between the simulated measured frequencies and the one obtained by the structural model.

The paper is divided into four main parts. Firstly, the sub-structuring strategy using OMR technique is presented. Secondly, the damage identification procedure using genetic algorithm is presented. Thirdly, numerical example is presented. Finally, a discussion of the result is given.

SUB-STRUCTURING REDUCTION STRATAGY

Since our interest is to identify damage in a specific region within a large structure, it will be determined as the main (master) sub-structure, while the other regions will be determined as the slave sub-structures. Our goal is to apply the identification procedure only to the physical parameters of the master while minimizing the monitoring efforts on the other sub-structures. It can be demonstrated in Figs. 1a and 1b.

In order to satisfy this goal, the OMR technique is adopted as the sub-structuring strategy. The DoFs obtained from the OMR indicate the optimum sensors location in the slave sub-structures. At the end, a reduced system is obtained, satisfying similarity of the internal forces on the interface between the reduced and the full model. Fig.1c illustrates the original and the condensed structural model according to the OMR. The identification procedure can then be applied only to the main structural parameters.

Mathematically specking, lets as consider a linear discrete un-damped structure, the equations of motion of the structure can be written as:

$$\mathbf{M}\ddot{u}(t)+\mathbf{K}u(t)=\mathbf{F}(t) \tag{1}$$

where **M** is the global mass matrix, **K** is the global stiffness matrix, **F**(t) is the global load vector, and **u**(t) is a displacement vector. Since, as was stated before, the idea is to separate the structure into main and attached parts, which interact with each other through the interface. The vector **u**(t) will be similarly partitioned into several sub-vectors, depending on the number of slave sub-structures attached to the main, $\mathbf{u}^T=\{\mathbf{u}_m, \mathbf{u}_{b1}, \mathbf{u}_{b2}, \mathbf{u}_{b3}, \mathbf{u}_{s1}, \mathbf{u}_{s2}, \mathbf{u}_{s3}\ldots\}$, representing the DOFs inside the main subsystem, on the interfaces ($\mathbf{u}_b$), and inside the slave subsystems ($\mathbf{u}_s$), respectively. Assuming that the slave sub-structures interact only through the main, the OMR procedure can be applied to each substructure separately, and the displacement vector can be considered as $\mathbf{u}^T=\{\mathbf{u}_m, \mathbf{u}_b, \mathbf{u}_s\}$ each time. Thus eq. (1) can be rewritten in a partitioned form as:

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{mb} & 0 \\ \mathbf{M}_{bm} & \mathbf{M}_{bb} & \mathbf{M}_{bs} \\ 0 & \mathbf{M}_{sb} & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_m \\ \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{mb} & 0 \\ \mathbf{K}_{bm} & \mathbf{K}_{bb} & \mathbf{K}_{bs} \\ 0 & \mathbf{K}_{sb} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_m \\ \mathbf{u}_b \\ \mathbf{u}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m \\ 0 \\ 0 \end{Bmatrix} \tag{2}$$

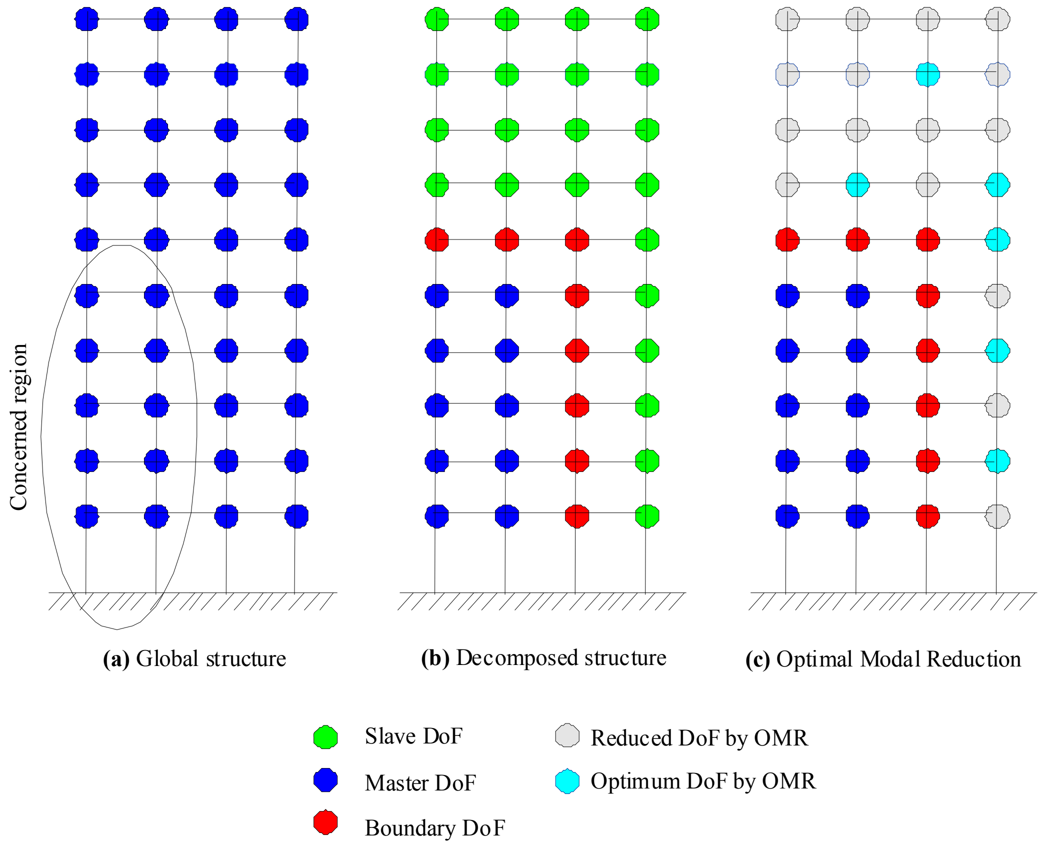

Figure 1. Original and condense structural model.

It is also assumed that the external vibration loading applied only to the main structure, as well as all non-zero initial conditions, if there are any. Since our goal is to reduce the number of sensors/DoFs in the slave substructure, having N_r<<N_s, were N_r and N_s are the number of DoFs in the slave substructure of the full and reduced models, respectively. The linear system is therefore reduced to the following one as:

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{mb} & 0 \\ \mathbf{M}_{bm} & m_{bb} & m_{br} \\ 0 & m_{rb} & m_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_m \\ \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_r \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{mb} & 0 \\ \mathbf{K}_{bm} & k_{bb} & k_{br} \\ 0 & k_{rb} & k_{rr} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_m \\ \mathbf{u}_b \\ \mathbf{u}_r \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

where $\mathbf{u}_r(t)$ is the N_r dimensional unknown vector associated with the reduced subsystem.

The reduction procedure is based on minimizing the differences between the internal forces in the interface of the full, $f_b(t)$, and the reduced systems, $\widetilde{f}_b(t)$, which are defined by the set of equations of the interface as follow:

For the full model:

$$\mathbf{M}_{bm}\ddot{\mathbf{u}}_m + \mathbf{K}_{bm}\mathbf{u}_m = -\mathbf{M}_{bb}\ddot{\mathbf{u}}_b - \mathbf{K}_{bb}\mathbf{u}_b - \mathbf{M}_{bs}\ddot{\mathbf{u}}_s - \mathbf{K}_{bs}\mathbf{u}_s = f_b(t) \tag{4}$$

and for the reduced model:

$$\mathbf{M}_{bm}\ddot{\mathbf{u}}_m + \mathbf{K}_{bm}\mathbf{u}_m = -m_{bb}\ddot{\mathbf{u}}_b - k_{bb}\mathbf{u}_b - m_{br}\ddot{\mathbf{u}}_r - k_{br}\mathbf{u}_r = \widetilde{f}_b(t) \tag{5}$$

The goal of the sub-structuring reduction strategy is to minimize the difference between these internal forces with respect to the sensors location in the slave sub-structures.

The OMR technique

The OMR technique defines the norm of the distance between these two vectors (eqs. 4 and 5) as:

$$\pi = \left\| f_b(t) - \widetilde{f}_b(t) \right\| = \frac{1}{T}\int_0^T \left| f_b(t) - \widetilde{f}_b(t) \right|^2 dt \tag{6}$$

where T is the time span of interest. Then it minimizes π with respect to the sensors location in the slave sub-structure. It can be interpolated by defining a modal weight for each set of sensor. The OMR procedure is given in details in Barbone et al. [11], Givoli et al. [6] and Tayeb and Givoli [12]. They found an upper bound for π as:

$$\pi \le \sum_{n=N_r+1}^{N_s} \left\| \mathbf{s}_n \mathbf{s}_n^T \right\| \tag{7}$$

where $\mathbf{s}_n$ was defined as follow:

$$\mathbf{s}_n = \omega_n \mathbf{M}_{bs} \phi_n - \frac{1}{\omega_n} \mathbf{K}_{bs} \phi_n \tag{8}$$

where ω_n and ϕ_n are the normalized eigenvalues and eigenvectors for the slave substructure's matrices. $\mathbf{s}_n$ can be interpolated as the modal weight of each set of DoFs in the slave sub-structures. After ranking the contribution of the mode according to $\left\| \mathbf{s}_n \mathbf{s}_n^T \right\|$, the location of the optimum sensors in the slave is obtained, and the mass and stiffness matrices of the reduced system (eq. 3) can be calculated as follow:

$$\begin{aligned}
&\mathbf{m}_{rr} = \mathbf{I}_r \ , \quad \mathbf{m}_{rb} = \mathbf{m}_{br}^T = \mathbf{\Phi}_r^T M_{sb} \\
&\mathbf{k}_{rr} = \mathbf{\Omega}_r^2 \ , \quad \mathbf{k}_{rb} = \mathbf{k}_{br}^T = \mathbf{\Phi}_r^T \mathbf{K}_{sb} \\
&\mathbf{m}_{bb} = \mathbf{M}_{bb} - \mathbf{M}_{bs}\mathbf{M}_{ss}^{-1}\mathbf{M}_{sb} + \mathbf{m}_{br}\mathbf{m}_{rr}^{-1}\mathbf{m}_{rb} \\
&\mathbf{k}_{bb} = \mathbf{K}_{bb} - \mathbf{K}_{bs}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sb} + \mathbf{k}_{br}\mathbf{k}_{rr}^{-1}\mathbf{k}_{rb}
\end{aligned} \tag{9}$$

where $\mathbf{I}_r$ is identity matrix (N_rxN_r), $\mathbf{\Omega}_r^2$ is a diagonal matrix (N_rxN_r) whose diagonal entries are the squared frequencies ω_n^2 of first N_r modes, and $\mathbf{\Phi}_r$ is matrix (N_sxN_r) whose columns are the first N_r eigenvectors ϕ_n.

After the sensor locations in the slave sub- structure were specified, and the reduced system was defined, the next step is to assess the stiffness distribution of the main sub-structure segements by an appropriate identification procedure.

IDENTIFICATION PROCEDURE

In this study, the identification procedure is based on minimization the differences between the *measured* modal dynamic response of the structure and the analytical one. The procedure defines a "Virtual Damaged Vector" (VDV) which infers the stiffness degradation at each of the elements of the main structure. Several optimization procedures can be applied using a desired cost function. In order to obtain a robust and global minimum the Genetic Algorithm (GA) is applied.

Genetic Algorithm

In the GA, a population of *Np* candidate solutions is first generated randomly. In our case, the candidate solutions are combination of the segments stiffness of the main structure (i.e. the population is composed of *Np* trial vectors, each of which represents a different combination of stiffness at the main structure, $\overline{VDV}_i = \{VDV_{i,1}, VDV_{i,2}, \ldots, VDV_{i,m}\}$; $VDV_{i,j}$ is the j component of vector $\overline{VDV}_i$, *m* is the number of segments in the main structure). The GA explores the search space by vector of various candidate solutions. At each iteration ("generation"), "mutant vectors" ($\overline{V}_i$) are formed by linear interpolation of trail vectors randomly selected from the population. A new generation of trial vectors ($\overline{U}_i$) is then formed by the "crossover" process, whereby the components of the mutant vectors are mixed with those of the trial vectors in the previous generation. If the trial vector yields a lower cost function value than the target vector, the trial vector replaces the target vector in the following generation. This last operation is called selection.

In this study, the cost function corresponded to the minimum of the differences between the measured and the analytical modal parameters, it is composed from the following two terms:

Modal Assurance Criteria (MAC):

$$Cost_1 = \sqrt{\frac{\left(\phi_{measured}^T \cdot \phi_{iteration}\right)^2}{\left|\phi_{iteration}^T \cdot \phi_{iteration}\right| \cdot \left|\phi_{measured}^T \cdot \phi_{measured}\right|}} - 1 \tag{10}$$

The norm of the frequencies:

$$Cost_2 = \frac{\left\|\omega_{measured} - \omega_{iteration}\right\|}{\left\|\omega_{measured}\right\|} \tag{11}$$

NUMERICAL EXAMPLES

The efficiency of the OMR in damage identification procedure is demonstrated through a multi-story building model, Fig. 2a. Assume that we would like to assess the stiffness distribution in-between the eighth and the thirteenth story. According to sub-structuring strategy, the structure will be divided into three sub-structures: two slaves and in-between the main sub-structure, see Fig. 2b. Applying OMR technique, the optimum sensors location, in the slaves, is specified. It is demonstrated in Fig. 2c. The identification procedure is then applied only to the segments in the main substructure.

This study is theoretical, but in order to simulate field conditions, random noises (5% random error) were added to the simulated structural response. Since each case is associated with a specific distribution of random noise. Repeated evaluation of 20 cases, each of which with a different random noise distribution, resulted in a population of solutions, for which the mean value of the identified stiffness is given in Fig. 3. In this case 25% stiffness reduction of the tenth story was considered. It is seen that good estimation for the stiffness distribution has been obtained.

In order to demonstrate the efficiency of the OMR reduced structural model, the identification procedure has been applied to the full structural model with the same numbers of sensors (installed evenly in the structure - in each even story) and with the same noise level. In this case the identification procedure absolutely failed to identify the stiffness distribution.

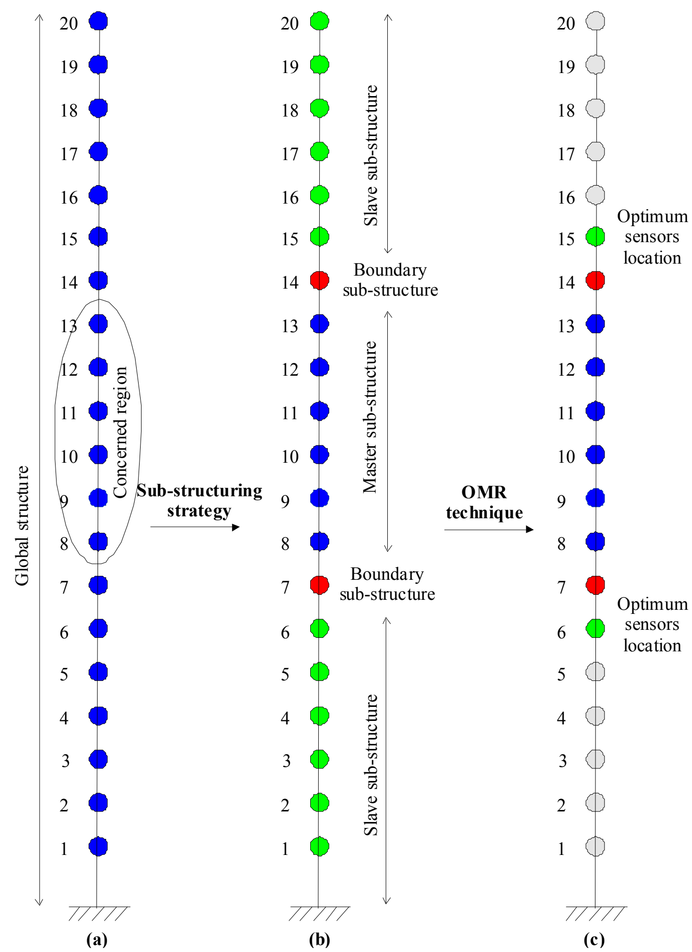

Figure 2. Structural model of 2th shear building.

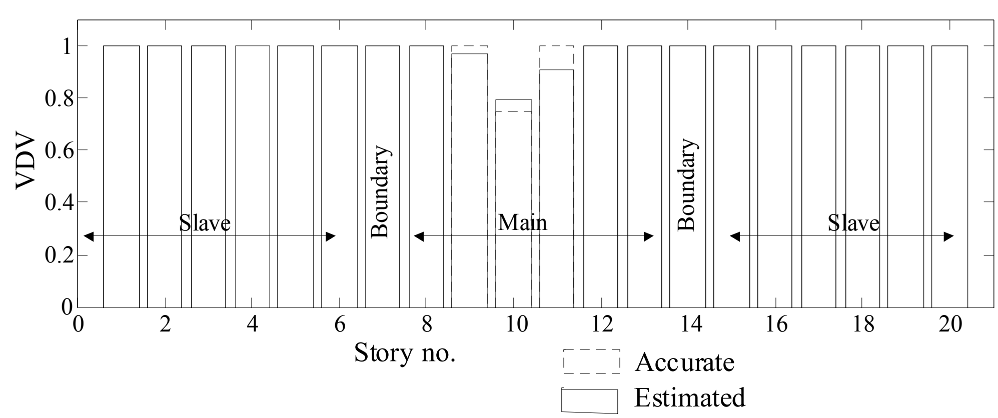

Figure 3. Estimated stiffness distribution (5% random noise).

SUMMARY AND CONCLUSIONS

The study demonstrated the efficiency of the OMR technique, as a sub-structuring strategy, in damage identification procedure through multi-story shear building model. A Genetic Algorithm (GA), based optimization procedure, is applied for minimizing the differences between the simulated measured modal dynamic properties and the analytical one. The effect of noisy signals and limited number of sensors are considered.

The reliability of the results, even in this simple structural model, is strongly depended on various factors, such as: the number of sensors (the total, in the master, in the slaves and the ratio between them), the noise level, the damaged severity and its location etc. Therefore, from these aspects, the study is not yet completed and further investigation should be performed in order to bring this concept to be fully realized in the structural damage identification procedure.

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