

An Algorithm for 3D Vibration Measurement Using One Laser Scanning Vibrometer

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ABSTRACT

3D vibration measurement is achieved using one laser scanning vibrometer(LSV) and Light Detection And Ranging(LIDAR) by moving the LSV to three arbitrary locations from the principle that vibration analysis based on the frequency domain is independent of the vibration signal based on time domain. The proposed algorithm has the same effect as using three sets of LSVs. It has an advantage of reducing the costs. Analytical approach of obtaining in-plane and out-of-plane vibration of surface is introduced using geometrical relations between three LSV coordinates and vibration measured at three different locations.

INTRODUCTION

Accelerometers are typically used to measure vibration in conventional tests of bulky machinery structures. However, these contact-type sensors have several disadvantages: their loading effect, which can affect the frequency response for a light and flexible structure; the tethering problem, which makes it hard to measure the vibration of a location apart from the sensors; and sensitivity to electromagnetic interference (EMI) effects, which makes it hard to measure vibration of power electronic components. Moreover, the use of contact-type sensors necessitates the use of multiple sensors when the vibration mode shape of a structure must be investigated[1].

These problems can be mitigated by using a non-contact laser Doppler vibrometer[2] (LDV). An LDV provides the velocity signal of an object using the Doppler frequency change that occurs due to interference between incident light and scattered light that is reflected from the surface of a moving object[3][4]. The LDV is

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used to detect delamination and disbond in composite plates. Using Doppler effects, LDVs are considered a technology that could rapidly and correctly measure vibration at a desired location. The vibration at multiple points can be easily measured by using a laser scanning vibrometer(LSV) in which laser beam is rotated by using an electric motor.

However, LSVs can only measure speeds parallel to the laser beam direction; i.e., they cannot detect vibration when the surface has vibration that is perpendicular to the laser beam direction. Therefore, three sets of LSVs are required in order to measure components of 3-dimensional (3D) vibration. The need for 3D measurements has increased since various industries have become interested in more accurate and high-speed measurements. For example, Bendel et al.[5] measured the vibration of power tools using a 3D scanning laser Doppler vibrometer (SLDV) to investigate vibration characteristics. In 2006, Miyashita et al.[6] proposed a method for measuring 3D vibration by calibrating the angles and locations of three sets of LDVs. Malley et al.[7] conducted research to search for landmines buried underground by acquiring the 3D vibration information obtained.

The use of three sets of LSVs, however, has a notable disadvantage of high cost. For 3D vibration measurement on an arbitrarily surface, it is possible to measure vibration of the surface at three different locations using one LSV pointing to the same measurement points. The proposed method has the same effect as using three sets of LSVs since a vibration analysis based on the frequency domain is independent of the vibration signal based on time domain measured simultaneously. Furthermore, the three different locations are easily obtained using a shape measurement device such as a laser scanner from the relative geometrical information between the shapes measured at different locations. Hence, the proposed method has an advantage of relieving the use of a mechanical frame to determine the relative geometrical relations employed in the conventional 3D vibration measurement as well as cost reduction.

PRINSIPLE OF PROPOSED 3D VIBRATION MEASUREMENT



Figure. 1. Comparison of (a) conventional 3D LSV and (b) proposed pseudo 3D LSV.

The conventional method for 3D vibration measurements is composed of three sets of LSVs and one set of shape measurement device such as LIDAR, as shown in Fig. 1(a). From the vibration and geometric shapes measured respectively from the LSVs

and LIDAR, in plane and out of plane vibration components on the measured surface are acquired. Fourier transforms of the time signal into frequency, out-of-plane, and in-plane vibration characteristics of an object are consequently performed. However, this method costs too much to be used for industrial applications. In addition, there are measurement constraints if the measurement locations are constrained by a mechanical frame. In order to mitigate the problem, this paper proposes an algorithm in which only one LSV with no mechanical frame is used for 3D vibration measurements.

The proposed system consists of one LSV and LIDAR (Fig. 1(b)). The LSV moves to three arbitrary locations to measure the vibration, while the LS is used to obtain the geometric relation between the LSV and an object. The proposed system has the same effect as using three sets of LSVs since a vibration analysis based on the frequency domain is independent of the vibration signal based on time domain measured simultaneously. As such, the proposed method can reduce equipment costs.



Figure. 2. Relationship between actual speed at arbitrary measurement point on object and speed measured from 3 different measurement locations.

Figure 2 presents the geometric relations of the coordinate systems $(x_1y_1z_1)$, $(x_2y_2z_2)$ and $(x_3y_3z_3)$ obtained by moving one LSV to three different locations. The origins of each coordinate system are attached to the scanning mirrors of the LSV. A local coordinate system $(x_Ly_Lz_L)$ is defined for each measurement point at the surface. The direction of the z_L axis of the local coordinate system $(x_Ly_Lz_L)$ is defined as being perpendicular to the surface of the measurement point, whereas the direction of the x_L axis is defined as being parallel to the x_1z_1 plane. Then, the direction of the y_L axis is defined as the outer production of the z_L and x_L axes. The vibration measured at the three measurement locations is defined as V_1 , V_2 , and V_3 respectively, and the angles between each axis of local coordinate system $(x_Ly_Lz_L)$ and V_1 , V_2 , and V_3 are defined as α_k , β_k , and γ_k ; where k=1,2,3. For simplicity, in Fig. 2 the angle between each axis of the first coordinate system $(x_1y_1z_1)$ and V_1 is expressed as α_1 , β_1 , and γ_1 .

Next, the vibration of the surface V_x , V_y , and V_z are obtained using, the angles (α_k , β_k , γ_k) and vibrations (V_1 , V_2 , V_3) as indicated in (2.1) [8].

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} \cos \alpha_1 & \cos \beta_1 & \cos \gamma_1 \\ \cos \alpha_2 & \cos \beta_2 & \cos \gamma_2 \\ \cos \alpha_3 & \cos \beta_3 & \cos \gamma_3 \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$
(2.1)

THE TRANSFORMATION MATRIX BETWEEN THE COORDINATE SYSTEMS

We propose a method for obtaining the transformation matrix between the coordinate systems defined in section. 2 by comparing the curved surface information measured using a LIDAR at each measurement location. Figure 3 shows the shape of the object measured at three measurement locations by using a LIDAR. The arrows are normal vectors that are perpendicular to the surface.



Figure. 3 Measurement shape and normal vectors at 3 different measurement locations.

The normal vectors of measurement points are obtained using the method of mean weight by areas of adjacent triangles (MWAAT)[9]. The surface measured at the different coordinate systems $(x_1y_1z_1), (x_2y_2z_2)$ and $(x_3y_3z_3)$ and the normal vectors are shown in Fig. 3. The normal vector set at every measurement point is then expressed as

$$(\mathbf{N})_{x_{k}y_{k}z_{k}} = \begin{bmatrix} \mathbf{n}_{1} & \mathbf{n}_{2} & \cdots & \mathbf{n}_{s} & \cdots & \mathbf{n}_{n} \end{bmatrix}_{x_{k}y_{k}z_{k}} = \begin{bmatrix} n_{1,x_{k}} & n_{2,x_{k}} & \cdots & n_{s,x_{k}} & \cdots & n_{n,x_{k}} \\ n_{1,y_{k}} & n_{2,y_{k}} & \cdots & n_{s,y_{k}} & \cdots & n_{n,y_{k}} \\ n_{1,z_{k}} & n_{2,z_{k}} & \cdots & n_{s,z_{k}} & \cdots & n_{n,z_{k}} \end{bmatrix}$$
(3.1)

where k =1, 2, 3.



Figure 5 Change of axis angle by the transformation matrix.

The normal vector sets obtained respectively at the coordinate systems $(x_2y_2z_2)$ and $(x_3y_3z_3)$ can be coincided with the normal vector set obtained at the coordinate systems $(x_1y_1z_1)$ if they are rotated at angles that are represented in the transformation matrix.

Suppose that the coordinate system $(x_2y_2z_2)$ is obtained by rotating the coordinate system $(x_1y_1z_1)$ by θ_{12} based on the y_1 axis and ϕ_{12} based on the x'_1 axis rotated from the x_1 axis by θ_{12} as shown in Fig. 5. The coordinate systems $(x_3y_3z_3)$ is similarly obtained by rotating the coordinate system $(x_1y_1z_1)$ by θ_{13} based on the y_1 axis and ϕ_{13} based on the x'_1 axis.

The normal vectors $(\mathbf{n}_s)_{x_2y_2z_2}$ and $(\mathbf{n}_s)_{x_3y_3z_3}$ represented in coordinate systems $(x_2y_2z_2)$, $(x_3y_3z_3)$ can be transformed into the normal vector $(\mathbf{n}_s)_{x_3y_3z_1}$ represented in the coordinate system $(x_1y_1z_1)$ using transformation matrices \mathbf{R}_{12} , \mathbf{R}_{13} and such

$$\left(\mathbf{n}_{s}\right)_{x_{l}y_{l}z_{l}} = \mathbf{R}_{lk} \times \left(\mathbf{n}_{s}\right)_{x_{k}y_{k}z_{k}} + \mathbf{e}_{s,lk}$$
(3.2)

where k=2, 3;

Similarly, suppose that the coordinate system $(x_L y_L z_L)$ is obtained by rotating the coordinate system $(x_1 y_1 z_1)$ by θ_{1L} based on the y_1 axis and ϕ_{1L} based on x'_1 axis. Then, the normal vector $(\mathbf{n}_s)_{x_L y_L z_L}$ represented in coordinate systems $(x_L y_L z_L)$ can be transformed into the normal vector $(\mathbf{n}_s)_{x_l y_1 z_1}$ represented in the coordinate system $(x_1 y_1 z_1)$ using transformation matrices \mathbf{R}_{1L} .

From the rotation angle between the coordinate systems, the transformation matrices \mathbf{R}_{12} , \mathbf{R}_{13} and \mathbf{R}_{1L} are calculated as

$$\mathbf{R}_{lk} = \begin{bmatrix} \cos^{2}\theta_{lk}(1-\cos\phi_{lk}) + \cos\phi_{lk} & \sin\theta_{lk}\sin\phi_{lk} & -\cos\theta_{lk}\sin\theta_{lk}(1-\cos\phi_{lk}) \\ -\sin\theta_{lk}\sin\phi_{lk} & \cos\phi_{k} & -\cos\theta_{lk}\sin\phi_{lk} \\ -\cos\theta_{lk}\sin\theta_{lk}(1-\cos\phi_{lk}) & \cos\theta_{lk}\sin\phi_{lk} & \sin^{2}\theta_{lk}(1-\cos\phi_{lk}) + \cos\phi_{lk} \end{bmatrix} \begin{bmatrix} \cos\theta_{lk} & 0 & \sin\theta_{lk} \\ 0 & 1 & 0 \\ -\sin\theta_{lk} & 0 & \cos\theta_{lk} \end{bmatrix}$$
(3.3)
where k=2, 3, L;

Note that $\mathbf{e}_{s,12}$, $\mathbf{e}_{s,13}$ and $\mathbf{e}_{s,1L}$ are errors associated with normal vectors experimentally obtained from the surface information using the laser scanner such as

$$\mathbf{e}_{s,1k} = \begin{bmatrix} e_{s,x_{1k}} \\ e_{s,y_{1k}} \\ e_{s,z_{1k}} \end{bmatrix}$$
(3.4)

where k = 2, 3, L.

Then, \mathbf{R}_{12} , \mathbf{R}_{13} and \mathbf{R}_{1L} are obtained from angles that minimize the sum of errors, $\varepsilon_{LSM,1k}$ in each measurement point using the least squares method, such that

$$\varepsilon_{LSM,1k} = \sum_{s=1}^{n} \left(\left| \mathbf{e}_{s,1k} \right|^{2} \right) = \sum_{s=1}^{n} \left(\left| \mathbf{n}_{s,x_{1}y_{1}z_{1}} - \mathbf{R}_{1k} \times \mathbf{n}_{s,x_{k}y_{k}z_{k}} \right|^{2} \right)$$
(3.5)

where k = 2, 3, L.

IN-PLANE AND OUT-OF-PLANE VIBRATION



Figure. 6 Direction vectors of laser beam.

Suppose that the coordinate of a measurement point P_s in three coordinate systems ($x_1y_1z_1$), ($x_2y_2z_2$) and ($x_3y_3z_3$) are obtained using a laser scanner as

 $P_1(p_{x_1}, p_{y_1}, p_{z_1})_{x_1y_1z_1}$, $P_2(p_{x_2}, p_{y_2}, p_{z_2})_{x_2y_2z_2}$ and $P_3(p_{x_3}, p_{y_3}, p_{z_3})_{x_3y_3z_3}$. Suppose that the vectors \mathbf{P}_1 , \mathbf{P}_2 and \mathbf{P}_3 are laser direction vectors pointing toward the origin of each coordinate system from the point P_s , respectively. They have magnitudes of distances from P_s to the origin of each coordinate system. Then, \mathbf{P}_1 , \mathbf{P}_2 , and \mathbf{P}_3 can be represented in vector forms as

$$\left(\mathbf{P}_{1}\right)_{x_{1}y_{1}z_{1}} = \begin{bmatrix} -p_{x_{1}} \\ -p_{y_{1}} \\ -p_{z_{1}} \end{bmatrix}, \left(\mathbf{P}_{2}\right)_{x_{2}y_{2}z_{2}} = \begin{bmatrix} -p_{x_{2}} \\ -p_{y_{2}} \\ -p_{z_{2}} \end{bmatrix}, \left(\mathbf{P}_{3}\right)_{x_{3}y_{3}z_{3}} = \begin{bmatrix} -p_{x_{3}} \\ -p_{y_{3}} \\ -p_{z_{3}} \end{bmatrix}$$
(4.1)

 $(\mathbf{P}_2)_{x_2y_2z_2}$ and $(\mathbf{P}_3)_{x_3y_3z_3}$ can be transformed into vectors represented in the coordinate system $(x_1y_1z_1)$ using transformation matrixes \mathbf{R}_{12} and \mathbf{R}_{13} such that

$$\left(\mathbf{P}_{2}\right)_{x_{1}y_{1}z_{1}} = \mathbf{R}_{12} \times \left(\mathbf{P}_{2}\right)_{x_{2}y_{2}z_{2}}$$
(4.2)

$$\left(\mathbf{P}_{3}\right)_{x_{1}y_{1}z_{1}} = \mathbf{R}_{13} \times \left(\mathbf{P}_{3}\right)_{x_{3}y_{3}z_{3}}$$
(4.3)

Knowing that \hat{i}_L , \hat{j}_L , \hat{k}_L are unit vectors on the axes x_L , y_L , z_L of local coordinate system $(x_L y_L z_L)$, the vectors $(\hat{i}_L)_{x_L y_L z_L}$, $(\hat{j}_L)_{x_L y_L z_L}$, $(\hat{k}_L)_{x_L y_L z_L}$ represented in the coordinate system $(x_L y_L z_L)$ are expressed as

$$(\hat{i}_{L})_{x_{L}y_{L}z_{L}} = \begin{bmatrix} 1\\0\\0 \end{bmatrix}, (\hat{j}_{L})_{x_{L}y_{L}z_{L}} = \begin{bmatrix} 0\\1\\0 \end{bmatrix}, (\hat{k}_{L})_{x_{L}y_{L}z_{L}} = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$$
(4.4)

 $(\hat{i}_L)_{x_L y_L z_L}, (\hat{j}_L)_{x_L y_L z_L}, (\hat{k}_L)_{x_L y_L z_L}$ also can be transformed into vectors represented in the coordinate system $(x_1 y_1 z_1)$ using transformation matrix \mathbf{R}_{1L} such that

$$(\hat{i}_L)_{x_L y_L z_1} = \mathbf{R}_{1L} \times (\hat{i}_L)_{x_L y_L z_L}$$

$$(4.5)$$

$$\left(\hat{j}_{L}\right)_{x_{L}y_{L}z_{L}} = \mathbf{R}_{1L} \times \left(\hat{j}_{L}\right)_{x_{L}y_{L}z_{L}}$$

$$(4.6)$$

$$\left(\hat{k}_{L}\right)_{x_{L}y_{L}z_{L}} = \mathbf{R}_{1L} \times \left(\hat{k}_{L}\right)_{x_{L}y_{L}z_{L}}$$
(4.7)

Then, we have the following:

$$\left(\hat{i}_{L}\right)_{x_{1}y_{1}z_{1}}\cdot\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}=\left|\left(\hat{i}_{L}\right)_{x_{1}y_{1}z_{1}}\right|\left|\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}\right|\cos\alpha_{k},$$
(4.8)

$$\left(\hat{j}_{L}\right)_{x_{1}y_{1}z_{1}}\cdot\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}=\left|\left(\hat{j}_{L}\right)_{x_{1}y_{1}z_{1}}\right|\left|\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}\right|\cos\beta_{k},\qquad(4.9)$$

$$\left(\hat{k}_{L}\right)_{x_{l}y_{l}z_{l}}\cdot\left(\mathbf{P}_{k}\right)_{x_{l}y_{l}z_{l}}=\left|\left(\hat{k}_{L}\right)_{x_{l}y_{l}z_{l}}\right|\left|\left(\mathbf{P}_{k}\right)_{x_{l}y_{l}z_{l}}\right|\cos\gamma_{k},\qquad(4.10)$$

where k=1,2,3 And finally:

$$\cos \alpha_{k} = \frac{\left(\hat{i}_{L}\right)_{x_{1}y_{1}z_{1}} \cdot \left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}}{\left|\left(\hat{i}_{L}\right)_{x_{1}y_{1}z_{1}}\right| \left|\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}\right|}, \qquad (4.11)$$

$$\cos \beta_{k} = \frac{\left(\hat{j}_{L}\right)_{x_{1}y_{1}z_{1}} \cdot \left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}}{\left|\left(\hat{j}_{L}\right)_{x_{1}y_{1}z_{1}}\right| \left|\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}\right|}, \qquad (4.12)$$

$$\cos \gamma_{k} = \frac{\left(\hat{k}_{L}\right)_{x_{1}y_{1}z_{1}} \cdot \left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}}{\left|\left(\hat{k}_{L}\right)_{x_{1}y_{1}z_{1}}\right| \left|\left(\mathbf{P}_{k}\right)_{x_{1}y_{1}z_{1}}\right|}.$$
(4.13)

where k=1,2,3

Then, V_x, V_y, V_z are obtained using Eq.(2.1)

CONCLUSION

In this paper, we introduced the algorithm for 3D vibration measurement of the surface using proposed system. The proposed system has advantage of reducing equipment costs. Using the geometrical relation obtained by LS and three vibrations obtained from each LSV measurement, in-plane and out-of-plane vibration of one arbitrary measurement point are measured. These procedures are applied on every measurement point. From the in-plane and out-of-plane vibrations of all measurement point, we can get several 3D mode-shapes that will be more helpful to understand property of the object.

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