Bayesian Experimental Design for Damage Detection in a Bolted Frame

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ABSTRACT

In order to design an appropriate structural health monitoring system, it is crucial to develop a methodology that incorporates the costs of each possible decision/action able to be taken with respect to each target damage state of the structure. To that end, this paper presents a framework based on Bayesian experimental design for choosing the optimal system for a given scenario. The cost parameters that govern the optimization are varied to represent different criteria that arise in different applications. Among these are situations where Type I error control is critical, where Type II error minimization is most important, and where minimal sensor count is critical. The proposed approach is then applied to data obtained from ultrasonic interrogation of a geometrically-complex, three-story frame structure with bolted joints.

INTRODUCTION

In the context of this study, the goal of implementing an SHM system is to minimize the total cost of operating the structure over its lifetime. All of the intended benefits of SHM—for example, improved reliability and life-safety, reduced downtime due to unexpected maintenance demands, the ability to adjust operation based on the state of the structure, lower costs through maintenance and performance optimization—may be expressed in terms of a cost-savings to the owner. Likewise, instrumenting the structure bears costs: the price of the sensors and data acquisition equipment, added weight, power consumption, and possible performance loss may be relevant depending on the application and particular SHM system used. Once these various costs have been identified and quantified, they may be used to design the SHM system which meets the goal of minimizing total cost to the user by incorporating them in a process known as Bayesian experimental design (BED).

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This study presents a case study in BED with the design of an ultrasonic guided wave structural health monitoring (UGWSHM) system for use on a bolted frame structure. UGWSHM utilizes elastic waves, typically excited by piezoelectric sensor-actuators, to monitor large regions of structures. These elastic waves are scattered upon interaction with geometric features in the structure (including damage) and reflected waves are then received at each sensor location [1]. By comparing to waveforms received at a reference state when the structure is known to be damage-free (known as "baselines"), differences in the scattered waveforms may be interpreted as indicating damage. An appropriate feature extraction process may then be selected to arrive at a decision of the structural state based on the information gathered. In this work, data from the bolted frame is fed into the BED process, and the optimal solutions over a range of system-related costs are studied.

EXPERIMENTAL PROCEDURE

The data required for running the BED optimization was obtained from UGW inspection of a bolted frame structure. The structure is shown in Figure **1**. It has three stories and 19 total elements, each made of 2-inch-wide by 1/8-inch-thick (5.1 cm by 3.2 mm) steel plates in 12 and 24 inch (30.5 and 61.0 cm) lengths. The elements are joined by steel angle brackets connected with two 1/4-inch-diameter (6.4 mm) bolts on each side.

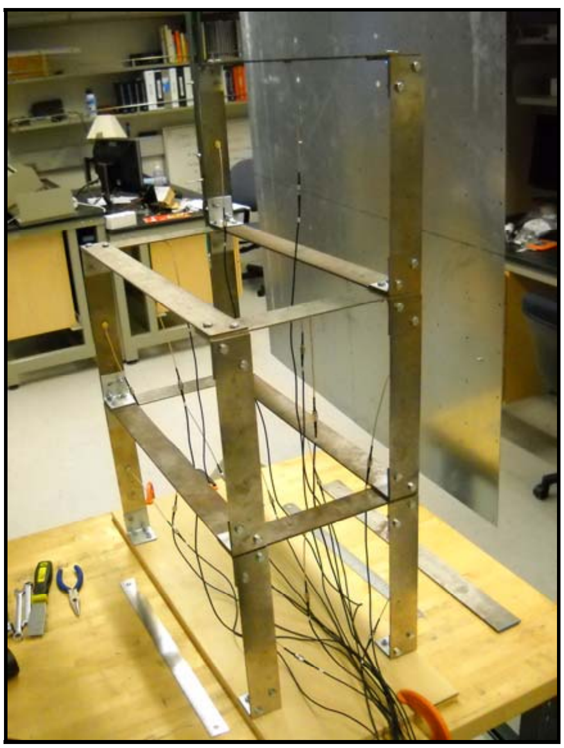

Figure 1. Bolted frame structure.

Sixteen PZT sensor-actuators were used to instrument the structure. The actuation signal used in this case was a Gaussian-modulated sinusoid with a center frequency of 135 kHz. At this frequency and in this structure, it was determined that the sensors predominantly produce waves in the first asymmetric mode (A_0). Received signals (in both pitch-catch and pulse-echo modes) were recorded on a National Instruments data acquisition system at 2.5 MHz.

Two damage modes were chosen for this test. First, connector preload loss was simulated by loosening several bolts, one at a time, in two increments. The second form of damage consisted of applying a magnet along one of the elements, which simulates any form of mid-element damage that reflects ultrasonic waves (e.g. cracking, corrosion). The differences between these forms of damage—location on the structure and the magnitude of the reflections (which are much higher for bolt loosening)—require different strategies for optimal detection.

FEATURE EXTRACTION

With raw data now recorded, the various waveforms must be converted into decisions on the damage state of the structure. This process is commonly known as feature extraction. The first step is to assess what part of the collected data is relevant to distinguishing damaged and undamaged cases. First, the signal is matched filtered by convolving the received signal with the actuation signal. This procedure limits the frequency content to the frequencies at which energy is input, under the implicit assumption that the data acquisition process is linear.

Next, a process known as optimal baseline subtraction (OBS) is applied to determine whether changes in the structure have occurred due to damage, while simultaneously providing some environmental compensation [2]. In order to use this technique, a database of baseline measurements must be taken when the structure is known to be in an undamaged condition. These measurements should span all of the environmental states the structure is expected to experience in operation. When the system is subsequently put in service, for each measurement taken, the nearest baseline (measured by some norm) is subtracted from it. The magnitude of the resulting residual should be near zero if the structure is undamaged, whereas damage is assumed to cause changes not found in the baseline set, causing the magnitude to rise substantially. Since the phase of the residual signal is highly uncertain, the final result is enveloped to remove the phase so that only incoherent detection is performed.

Finally, each waveform must be reduced to a scalar value that can be compared to a threshold. In this work, the energy metric (sum-square of the signal) will be used, although the BED procedure is independent of the type of feature selected. (In fact, BED may be used to determine the optimal feature.) Next, we compute receiver operating characteristics (ROC) for each of the tests. The ROC is a way of comparing the tradeoff between Type I error (predicting damage when damage is not present) and Type II error (predicting no damage when damage is present) [3]. Type I error is also known as false alarm, whereas Type II error is known as missed detection. When classifying measurements as either damaged or undamaged, a decision threshold value must be selected, above which the system classifies readings as damaged. For any group of feature values which are not completely separable (the vast majority of real applications), there will arise some rate of Type I and Type II error depending on the threshold selected. As part of selecting the optimal SHM system design, the BED algorithm will choose the threshold value which provides the optimal tradeoff for the parameters considered.

BAYESIAN EXPERIMENTAL DESIGN THEORY

In order to design an SHM system optimally for a particular situation, the Bayesian experimental design (BED) technique is exploited [4, 5]. The criterion for designing the best SHM system from the standpoint of Bayes cost may be expressed as:

$$\min_{e}\left(\sum_{i,j}\mathrm{C}^{\mathrm{d}}\left(d_i,\theta_j\right)\mathrm{P}\left(d_i\,|\,\theta_j,e\right)\mathrm{P}\left(\theta_j\right)+\mathrm{C}^{\mathrm{e}}\left(e\right)\right)$$

The symbol *e* refers to the "experiment", or in this case the SHM system. This notation may take into account a wide variety of design parameters, including the type, number, and location of sensors, the data acquisition used, and elements of the feature extraction process (such as the decision threshold). The sum is over all possible decisions d_i taken in response to the data gathered and all possible states of the structure, θ_j. The term $\mathrm{C}^{\mathrm{d}}\left(d_i,\theta_j\right)$ is therefore the cost of making a decision given the true state of the structure. The likelihood function $\mathrm{P}\left(d_i\,|\,\theta_j,e\right)$ is the probability that the system will make decision d_i given the actual state of the structure θ_j and the SHM system used to assess that true state (*e*). This probability is estimated from the probability of detection and the probability of false alarm from the tests conducted (data-driven) or from validated models (model-driven). Finally, the term $\mathrm{P}\left(\theta_j\right)$ is the prior probability of each state occurring, and $\mathrm{C}^{\mathrm{e}}\left(e\right)$ is the cost of implementing the SHM system.

System Definition

The first step to applying the BED approach to a problem is to identify the target states of the structure to be assessed [6]. In this case, the structure is considered to be in an undamaged state, to have sustained bolt loosening damage, or to have sustained mid-element magnet damage. Next, the possible actions that may be taken in response to information on the structure's state must be determined. In this case, we will simply consider that when the SHM system detects damage, we shall inspect and repair, whereas we shall do nothing if it is undamaged. The likelihood function is estimated from the test data – that is, the probabilities for a given system design and damage state are found from the percentage of the test cases that are classified in each damage category. Finally, the prior probabilities that each damage state will occur must be determined. Again, because this is a lab structure with no intrinsic prior history of failure, loading, etc., the prior probabilities were taken to be equal for bolt and magnet damage modes.

BAYESIAN EXPERIMENTAL DESIGN RESULTS

If all of the constraints of a problem are known, the BED procedure can be used to optimize the SHM system in a number of different ways. For example, the optimal number of sensors can be determined based on the cost per sensor and the various decision costs. Figure **2** depicts the lowest (normalized) system cost for each number of sensors from two to sixteen. Each plot represents different decision cost ratios relative to the sensor cost.

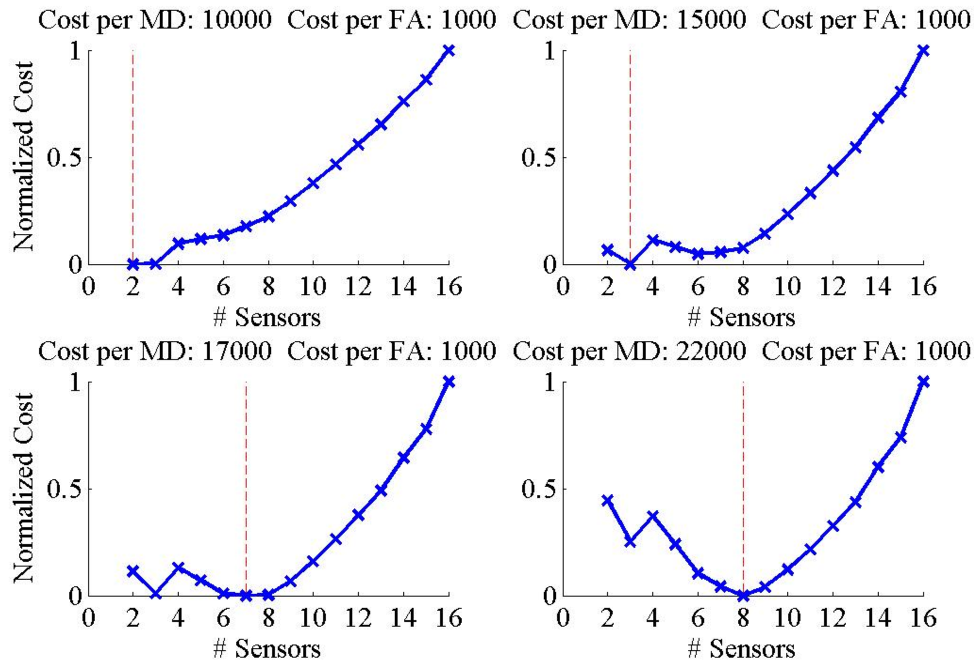

Figure 2. Normalized system cost vs. number of sensors used.

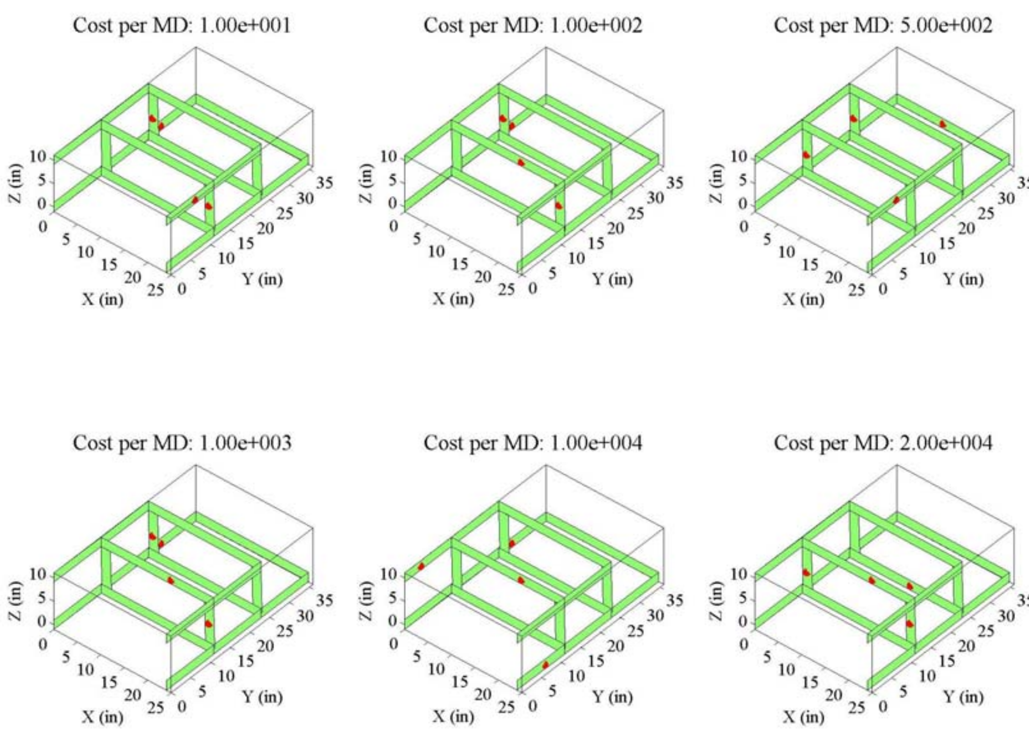

Figure 3. Sensor locations for different missed detection costs. Fixed false alarm cost = 1.

By contrast, if for weight, space, or other constraints the system is limited to a certain number of sensors, the BED procedure can localize them based on the relative costs of false alarm and missed detection for each type of damage. Figure **3** shows examples from the bolted frame structure when constrained to have only four sensors. Four sensors is extremely sparse for the size of the structure being monitored, which serves to highlight the trade-off as the ratio between false alarm cost and missed detection cost changes. When the missed detection cost is relatively low, as in the top-left figure, the system resorts to the most conservative arrangement possible. That is, the sensors tend to cluster in smaller regions, achieving very accurate local prediction by sacrificing coverage area in order to minimize false alarms. By contrast, when the missed detection cost is relatively high, as in the bottom-right figure, the sensors tend to spread out to maximize coverage and minimize the areas where damage might escape detection.

The trade-offs can become even more explicit by looking at the receiver operating characteristics directly. Figure **4** presents results considering only the magnet damage case with exactly four sensors on the structure. Each line on the plot represents the ROC of the sensor arrangement that was found to be optimal for each missed detection cost value, with the cost of false alarm held constant at 1. The square markers indicate the point on those ROC curves that provided the optimal performance. As the missed detection cost increases, the system becomes more biased towards solutions that have a better detection rate as well as the accompanying higher false alarm rate. This is manifested graphically in that the square markers continue to move up and right. Furthermore, it is clear that the markers in this case are always closest to the top left in their particular region—that is, the BED algorithm is selecting sensor arrangements that are *locally* optimal (in terms of the ROC) for the false alarm to missed detection ratio specified. Similar results may be observed in Figure **5** for the bolt damage case, although due to superior detectability of that damage type, there is not as much variability in the different solutions.

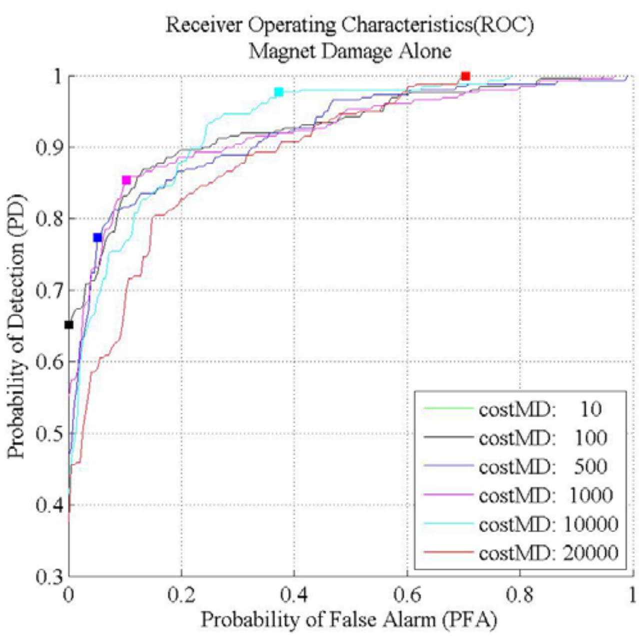

Figure 4. ROC plots for four sensor arrangements, magnet damage only. Squares indicate the optimal points selected by the BED algorithm.

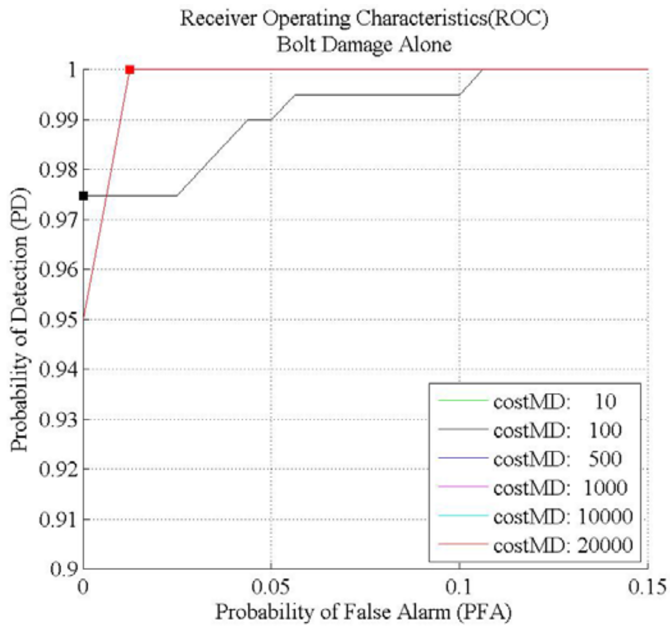

Figure 5. ROC plots for four sensor arrangements, bolt damage only. Squares indicate the optimal points selected by the BED algorithm.

However, because the two forms of damage are simultaneously possible in the structure, it is more realistic to look at the sensor arrangements which minimize the combined cost for both forms of damage. The resulting ROC plots are shown in Figure **6**. It is clear from the right-hand plot that the chosen points on the ROC curve are no longer optimal for each case independently. Instead, because detecting the each form of damage requires a different strategy, the optimal solution consists of a compromise.

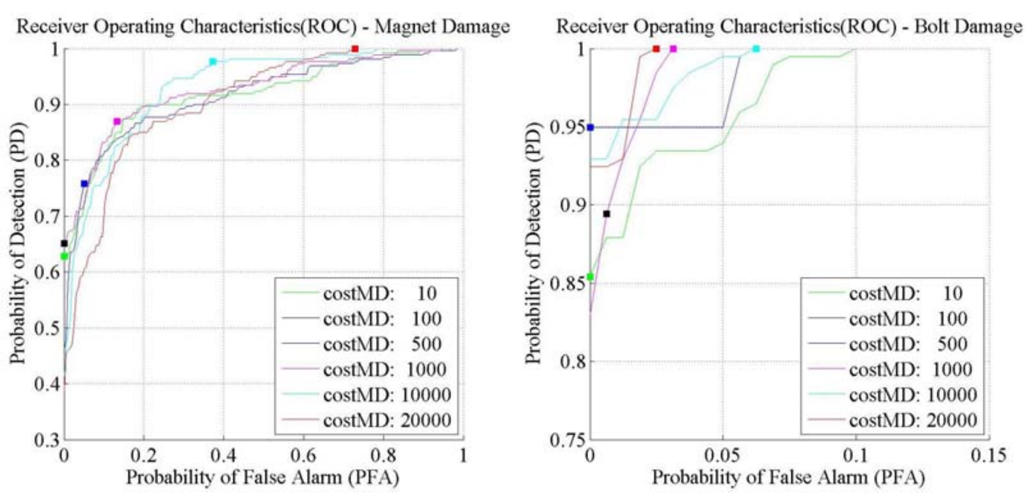

Figure 6. ROC plots for four sensor arrangements. Squares indicate the optimal points selected by the BED algorithm.

CONCLUSIONS

BED provides a useful tool for balancing the various trade-offs that arise in the design of an SHM system. Through a case study applied to UGWSHM in a bolted frame, the principles of system selection through the BED process have been demonstrated. First, the BED process was used to determine the optimal number of sensors to use for the SHM system based on the costs of the sensors and the costs of incorrectly assessing the damage. In general, more sensors are used when the decision costs increase relative to the sensor cost. Next, it was assumed that the sensor system is limited to four sensors, and the optimal arrangements of those sensors based on the costs of Type I and Type II error were plotted. By visual inspection, it is clear that the system prefers smaller clusters of sensors when Type I error cost is high, as opposed to maximizing coverage when Type II error cost is high. Through ROC analysis, we have shown that the system selects the optimal point on the ROC curve based on the cost ratios and the types of damage considered.

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