

# Substructure Isolation Method for Online Local Damage Identification Using Time Series

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## ABSTRACT

This paper proposes a Substructure Isolation Method based on time series (SIM-TM) of measured local response and intended for local online monitoring of substructures. The method consists of two key steps: (numerical) construction of the isolated substructure, and local identification. The isolated substructure is an independent virtual structure, which is separated from the global structure with virtual supports placed in their interface. In the first step, the response of the isolated substructure is constructed by linear combinations of sub-time series of the measured local responses. Then, natural frequencies of the isolated substructure are identified based on the constructed response and used for local identification. The method has no requirements on the initial state of the structure. The isolation can be carried out time section by time section using the successive fragments of the measured responses, so that the approach can be used for online monitoring. A numerical frame model is used to verify the proposed online monitoring method.

## INTRODUCTION

Structural Health Monitoring (SHM) is an important field of research that contributes to the safety and reliability of civil engineering structures. However, structures in civil engineering are often large and complex. Due to the limitation of sensor placement, and the insensitivity to local damage, such structures are difficult to be identified globally and precisely [1]. Sometimes, only parts or local components are crucial and should be concerned for monitoring, like supports of bridges, bottom pillars

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of high buildings, etc. Therefore, substructuring methods are practical in such application, as they identify local damages accurately using only a few sensors placed on the substructure.

The substructuring methods are relatively widely researched. Currently, substructures can be identified using Kalman filter [2,3], auto-regressive and moving average model (ARMAX) [4], genetic algorithms [5], Substructure Isolation Method (SIM) [6,7], etc. In real application, some local substructure is crucial and important, and they need to be monitored online. The present literatures often use least-square estimation [8,9] and adaptive extended Kalman filter [10] to track the structural damage online in time domain.

This paper presents a Substructure Isolation Method (SIM) [6,7] using time series of local measured responses, which can be used for online identification of substructural damage. The SIM method first isolates the substructure from the global structure into an independent and virtual structure, and then identifies the isolated substructure using a model-based method [11,12]. A numerical frame model is used for verification.

## ONLINE ISOLATION AND IDENTIFICATION

Damages of a substructure can be identified quickly and efficiently based on the natural frequencies of the isolated substructure, which is constructed using measured local substructural responses. These responses can be due to any general excitation, such as random excitation, impact excitation, which makes the proposed method well-suited for online substructure identification. Figure 1 shows the flow chart of the online identification.

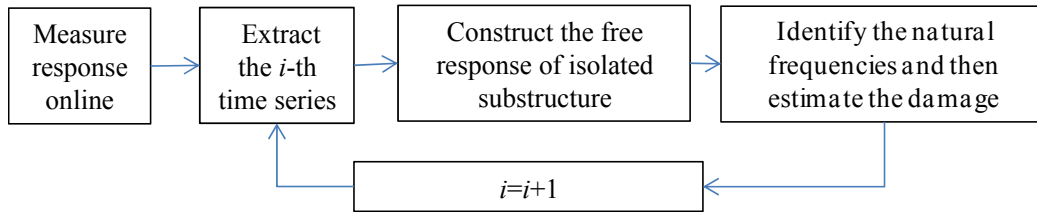


Figure 1. The flow chart of the online substructure identification.

## The SIM Method Based on Sub-time Series

The free responses of the isolated substructure can be computed using the isolation formula, see Equation (1),

$$\mathbf{u}_Y = \mathbf{d} - \mathbf{CA}^+\mathbf{b} \quad (1)$$

where the “+” superscript denotes the pseudo-inverse. The matrices  $\mathbf{C}$ ,  $\mathbf{A}$  and the vectors  $\mathbf{d}$ ,  $\mathbf{b}$  are constructed using the measured local substructural responses, which is explained in the following.

Assume the substructural boundary has  $l$  Degrees of Freedom (Dofs). Then  $l$  sensors denoted as  $b_1, b_2, \dots, b_l$  are placed on the boundary to measure all the boundary responses, which are denoted as  $\{x_1(t), x_2(t), \dots, x_l(t)\}$ . Furthermore,  $n$  sensors are placed inside the substructure, denoted as sensor 1, 2,  $\dots$ ,  $n$ , and the corresponding measured responses are  $\{y_1(t), y_2(t), \dots, y_l(t)\}$ . Let  $f_s$  be the sampling frequency, so

that the sampling time interval  $\Delta t$  is equal to  $1/f_s$  and the sampling time instances are  $t_i = i\Delta t$  for  $i=1,2,3,\dots$ . Collect  $k$  sub-series, each of length  $w$ , of the measured responses. The  $i$ th sub-series ( $i=1, 2, \dots, k$ ) is obtained by sampling the responses in time instances  $\{t_1^i, t_2^i, \dots, t_w^i\}$ , where  $t_j^i$  is the  $j$ th time step of the  $i$ th sub-series. Let  $\mathbf{X}_{ji} = \{x_j(t_1^i), x_j(t_2^i), \dots, x_j(t_w^i)\}$  be the  $i$ th sub-series of the  $j$ th boundary sensor response, see Figure 2; similarly, let  $\mathbf{Y}_{ji} = \{y_j(t_1^i), y_j(t_2^i), \dots, y_j(t_w^i)\}$  be the  $i$ th sub-series of the  $j$ th inner sensor response. Then in each sub-series all the boundary responses are rearranged into a single vector  $\mathbf{X}_i = \{\mathbf{X}_{1j}, \mathbf{X}_{2j}, \dots, \mathbf{X}_{lj}\}$ , where  $\mathbf{X}_i$  is a column vector with the length of  $lw$ , and  $i=1,2,\dots,k$ , see Figure 3. Similarly, in each sub-series all the inner responses are rearranged into a single vector  $\mathbf{Y}_i = \{\mathbf{Y}_{1j}, \mathbf{Y}_{2j}, \dots, \mathbf{Y}_{lj}\}$ , where  $\mathbf{Y}_i$  is a column vector with the dimension of  $nw$ ,  $i=1,2,\dots,k$ . Finally,  $\mathbf{A} = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_{k-1}]$ ,  $\mathbf{b} = [\mathbf{X}_k]$ ,  $\mathbf{A} = [\mathbf{Y}_1, \mathbf{Y}_2, \dots, \mathbf{Y}_{k-1}]$  and  $\mathbf{d} = [\mathbf{Y}_k]$  can be constructed.

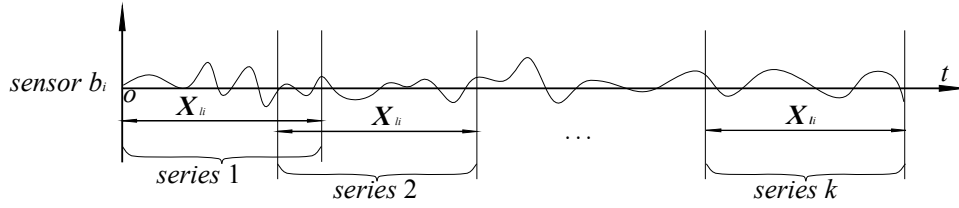


Figure 2. The sub-series selection of the responses of the substructural boundary sensor  $b_i$ .

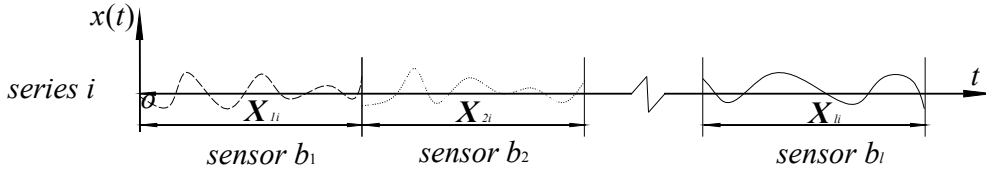


Figure 3. The arrangement of series  $i$  of the boundary responses into a single vector  $\mathbf{X}_i$ .

## Substructure Identification

For substructure identification, first the free response of the isolated substructure is constructed and then the identification is performed via comparison the identified modes of the isolated substructure with the modes estimated using its finite element model under given damage extent. The identified modes are obtained from the constructed free responses. Determination of some parameters is important for the identification accuracy, such as the sampling frequency  $f_s$ , the length of the sub-time series  $w$ , the number of the sub-time series, and the time delay of the adjacent sub-time series  $\Delta t_s$ . Theoretically, the sampling frequency should satisfy  $f_s \geq 2f_{max}$  and the length of sub-time series  $w$  should satisfy the relation  $w > 2f_s/f_1$ , where  $f_{max}$  is the largest involved natural frequency of the isolated substructure, and  $f_1$  is its first natural frequency. The number of the sub-time series  $k$  should allow a combination of the responses of the  $l$  boundary sensors in  $w$  time steps to vanish, so that  $k$  should satisfy the

relation  $k \geq lw$ . For simplicity, let the time delay  $\Delta t_s$  of any adjacent sub-time series be the same, where  $\Delta t_s = t_1^{i+1} - t_1^i$  for  $i=1,2,3,\dots$  and  $h=1$  or  $2$ .

After the free response of the isolated substructure is constructed, its natural frequencies can be identified by Eigensystem Realization Algorithm (ERA) and then used for identification of the substructure.

## NUMERICAL EXAMPLE

A numerical model of a 3 floor frame (Figure 4) is taken to test the proposed method for online identification. The members are numbered as shown in Figure 4. Each floor is 0.5 m high, and each span is 0.5 m wide. The density is  $7850 \text{ kg/m}^3$ , and its Young's modulus is 210 GPa. The cross-section of pillars and beams is  $6\text{mm} \times 50\text{mm}$ . The first and the second order damping ratio are set as 1%. The right hand pillar in the first floor is chosen as the substructure to be monitored online, see Figure 4.

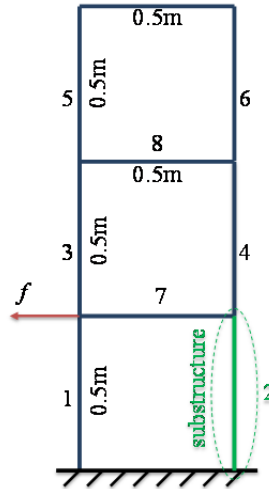


Figure 4. Frame structure model.

### The Boundary of the Substructure

There are three Dofs on the boundary: the axial displacement  $u_2$ , the horizontal displacement  $u_1$ , and the rotation  $\theta$  (Figure 5a). Since the axial displacement  $u_2$  is very slight,  $u_1$  and  $\theta$  are the important Dofs of the boundary. Therefore only horizontal response and rotation response need to be measured. As it is well known, it is not easy to measure the rotation  $\theta$ , and acceleration is the most popular measurement adopted in real applications. So the acceleration  $a_2$  is measured here instead of the rotation  $\theta$ , see Figure 5b, where  $a_2 = \ddot{u}_2 + \ddot{\theta}l$ ,  $l=70\text{mm}$ . The horizontal accelerations  $a_1$  and  $a_3$  are also measured. When the boundary acceleration response  $a_1$  and  $a_2$  are combined to zeros using the SIM method, then the related sensors can be equivalently converted to virtual supports, see Figure 5c. As a result, the substructure is isolated from the global structure into an independent isolated substructure. It can be computed that the first natural frequency of the intact isolated substructure is 127.77Hz via its finite element model.

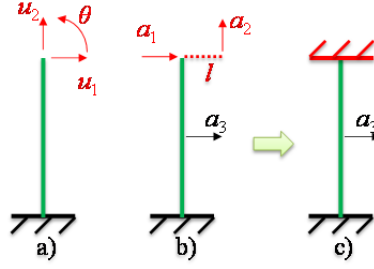


Figure 5. (a) the boundary of the substructure; (b) sensor placement; (c) isolated substructure.

### Online Identification of the Substructure Damage

Assume that the structural damages are changing with time and each member has one damage extent, which is the ratio of its actual stiffness to the original stiffness. Figure 6 shows the nine damage extents. Each bar shows the assumed time-history of the damage extent in the measured time interval, which is 11s. The damage extent of the 2nd pillar changes as shown in Equation (2)

$$\mu(t) = \begin{cases} 0.8; & \text{if } t \leq 2.2s \\ 0.8 - 2(t - 7.7)/55; & \text{if } 2.2s < t \leq 7.7s \\ 0.6; & \text{if } 7.7s < t \leq 11s \end{cases} \quad (2)$$

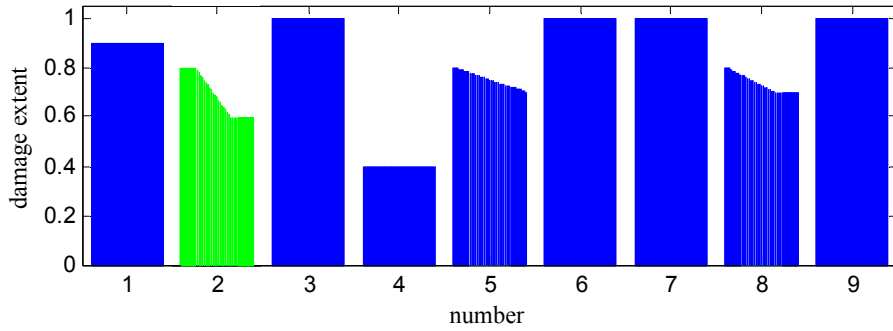


Figure 6. The time-histories of the damage extents of the structural members.

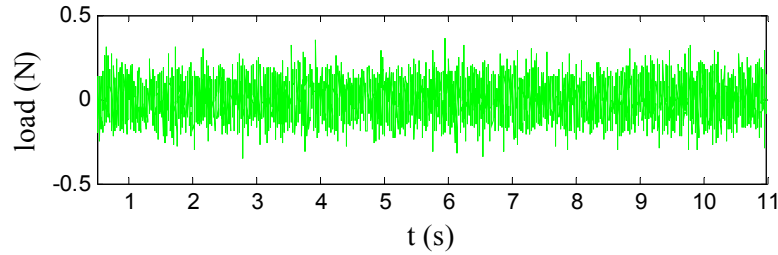


Figure 7. White noise excitation.

In order to identify the substructure, a white noise excitation (Figure 7) is applied to the second floor, see Figure 4. The sampling frequency  $f_s$  is 5000Hz. The three acceleration responses  $a_1$ – $a_3$  are simulated and respectively shown in Figure 8.

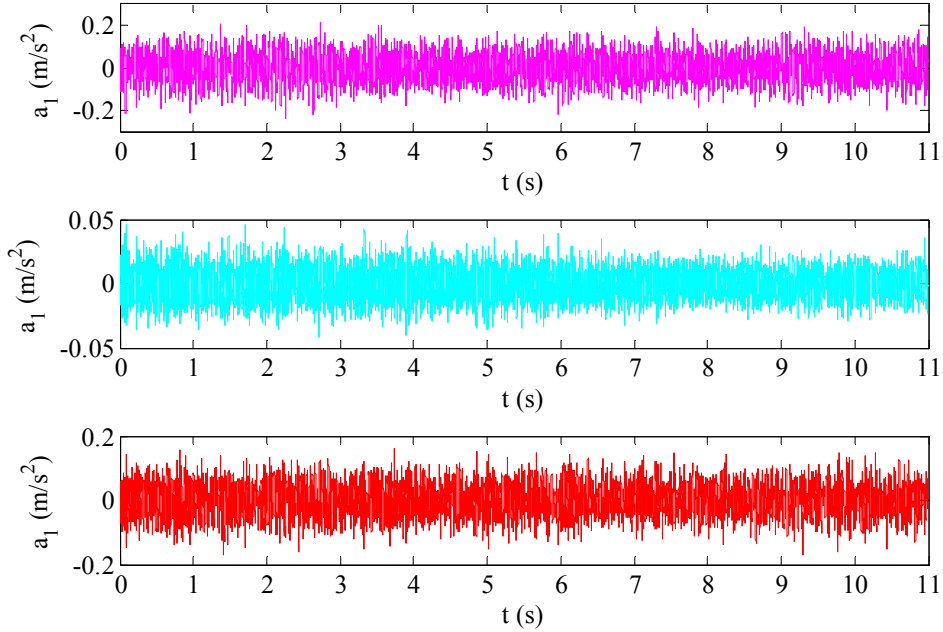


Figure 8. The measured (simulated) acceleration responses.

There is only one substructural member to be monitored, so one natural frequency is enough for identification. As mentioned above, the computed first natural frequency of the intact isolated substructure is  $\omega_{intact}=127.77\text{Hz}$ . In order to satisfy the requirements on the parameters of the analyzed time series, the length of sub-series in each stage  $w$  is set as 100 time steps, the number of the sub-time series  $k$  is 220, and the time delay of any two adjacent sub-time series is  $\Delta t_s = \Delta t$ .

For example, if the damage of the substructure in time 0.22s is to be monitored, then the time series of the responses from 0.198s to 0.242s are selected. 220 sub-series with the length of 0.01s are picked from the time series. The matrices **A**, **C** and the vectors **b**, **d** are constructed using the selected 220 sub-series. The corresponding free responses of the isolated substructure can be constructed using Equation (1). Figure 9 shows the free responses constructed in the mentioned time interval considering 0% and 5% noise pollution respectively. The identified natural frequency without noise is  $\omega_{0\%}=115.74\text{Hz}$ , and with 5% noise pollution it is  $\omega_{5\%}=114.80\text{Hz}$ . The damage extent of the substructure is estimated by the square of ratio between the identified natural frequency and the intact frequency, so that the estimated damage extent without noise  $\mu_{0\%} = (115.74/127.77)^2=0.82$ , and the estimated damage extent with 5% noise  $\mu_{5\%} = (114.80/127.77)^2=0.81$ . The real assumed damage extent of the substructure in 0.22s is 0.8, so the identification in the two cases are both accurate.

By repeating the above procedure, the substructure damage is identified online for every 0.01s. Figure 10 and Figure 11 show respectively the results of online monitoring of the substructure damage. In Figure 10, the identification is performed without

considering the noise, and the accuracy is quite good, and Figure 11 shows the identified results considering 5% Gauss noise, which yields results that are still acceptable.

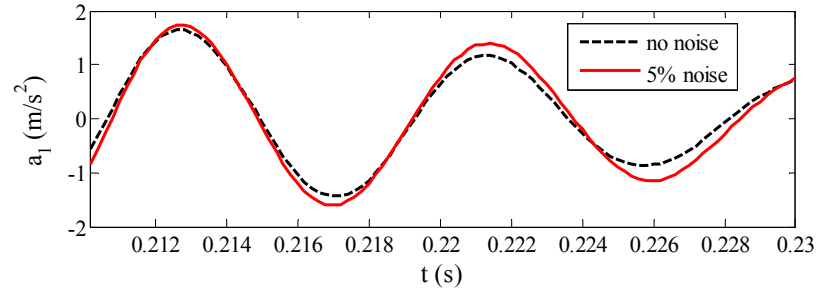


Figure 9. The construed free responses of isolated substructure.

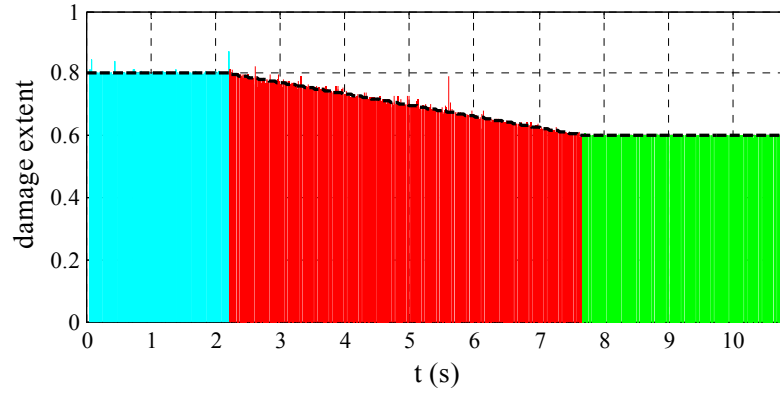


Figure 10. The identified time history of the substructure damage extent without measurement noise.

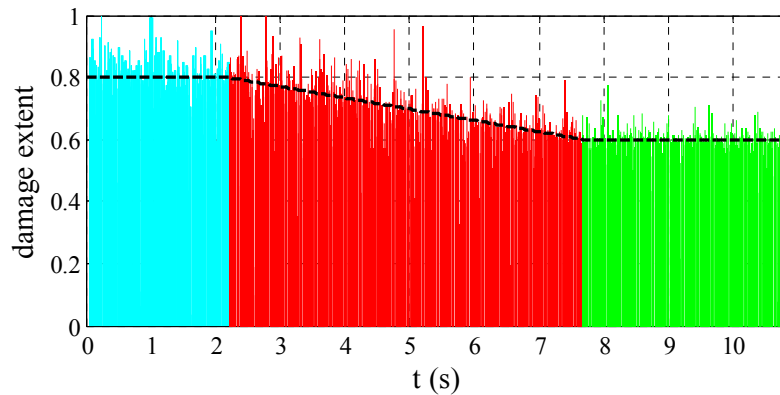


Figure 11. The identified time history of the substructure damage extent with 5% measurement noise.

## CONCLUSION

This paper develops the substructure isolation method for online local monitoring using time series of the measured response. A numerical simulation of a frame model is performed to verify the efficiency and accuracy of the proposed method. Experimental verification is in progress.

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