Numerical Studies of a Damage Detection Method for Beam Structures Based on Local Flexibility and Modal Macro-Strain

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ABSTRACT

The vibration-based global damage detection methods try to extract modal parameters from vibration signals as the main structural features and then apply these features to perform damage diagnosis. For a beam structure, the vibration signals are usually lateral acceleration, velocity or displacement. As a result, the extracted mode shapes are "lateral displacement" mode shapes. In this study, the "rotatory displacement" mode shapes were extracted from the macro-strain vibration signals. These rotatory displacement mode shapes were employed to detect damage of a beam structure utilizing the local flexibility method. The proposed method was verified by numerical studies of a simply supported beam. The finite element model was constructed using the ANSYS software with solid elements. The exact mode shapes and natural frequencies of the intact and damaged cases were obtained from modal analysis of the finite element model. The effects of the number of modes, damage locations and noise in the modal parameters on damage detection results were discussed in the numerical studies. The results illustrate potential feasibility of the proposed idea and the potential advantage of utilizing macro-strain mode shapes over the lateral displacement mode shapes in noisy conditions. However, further experimental research is necessary to verify the applicability of the proposed approach to real structures.

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INTRODUCTION

Due to the increasing demand of maintaining performance, reliability, and cost-effectiveness in civil, mechanical, and aerospace communities, the ability to promptly and accurately detect, localize, and quantify structural damage has become an important factor. The vibration-based technique which detects damage in a structure from changes in global dynamic properties is one of the promising fields in structural damage detection. Relating structural damage detection techniques have attracted much attention in recent years, and many approaches have been developed.

Recently, Abdo and Hori (2002) demonstrated the usefulness of the rotation of mode shape as a more sensitive diagnostic parameter than the displacement mode shape for damage localization in flexural structures. However, the application of the rotation of mode shape is only theoretical and numerical. In addition, Li and Wu (2007) illustrated the feasibility of damage detection algorithms on the basis of dynamic macro-strain measurements from long-gauge FBG sensors. Because the rotary mode shapes can be obtained from the macro-strain mode shapes, the experimental application of the rotary mode shapes for damage detection seems possible.

On the other hand, Toksoy and Aktan(1994) first tried to detect damage locations based on structural flexibility matrices of a beam structure. However, the damage detection algorithms based on flexibility matrices lack a solid theoretical background until the damage location vector method which can locate damage was developed by Bernal (2002). Reynders and De Roeck (2010) further developed the local flexibility method with a robust theoretical background to not only detect damage locations but also damage extents. The local flexibility method utilizes flexibility matrices of a beam structure before and after damage constructed by lateral displacement mode shapes. Combined with corresponding load configurations which cause strain and stress fields within a local region of the beam structure, the damage extent of the local region can be estimated.

In this study, the idea to utilize macro-strain measurement via the local flexibility method to perform damage localization and quantification of a beam structure is proposed. Numerical studies considering practical issues including the limited number of structural modes and noise effect in the modal parameters were investigated. The numerical results was also compared to the one utilizing the local flexibility method based on lateral displacement mode shapes.

METHODOLOGY

The local flexibility method which not only localizes but also quantifies the damage of a structure has been developed by Reynders and De Roeck (2010). Consider a structure with volume $\mho$ and boundary Γ which is subjected to the Dirichlet boundary conditions $\boldsymbol{x} = \bar{\boldsymbol{x}}$ along part of the boundary. A first load system $\boldsymbol{f}^1$ is applied at a limited number of l DOFs where response can be measured. The first load system is chosen such that the induced stress field $\boldsymbol{\sigma}^1$: (1) can be calculated from the loading without knowledge of the structure's stiffness and (2) consists of nonzero stresses in a small volume $\mho_p$ only. The stiffness within $\mho_p$ is assumed constant.

Based on the virtual work principle:

$$\int_{\mho} \rho \boldsymbol{b}^T \delta \boldsymbol{x} d\mho + \int_{\Gamma} \boldsymbol{t}^T \delta \boldsymbol{x} d\Gamma = \int_{\mho} \boldsymbol{\sigma}^T \delta \boldsymbol{\varepsilon} d\mho \tag{1}$$

where $\boldsymbol{b} \in \mathbb{R}^{3\times1}$ is the vector with body forces, $\boldsymbol{t} \in \mathbb{R}^{6\times1}$ the vector with applied tractions, $\boldsymbol{\sigma} \in \mathbb{R}^{6\times1}$ the corresponding stress vector, $\delta \boldsymbol{x} \in \mathbb{R}^{3\times1}$ a virtual displacement field that obeys the Dirichlet boundary conditions and $\delta \boldsymbol{\varepsilon} \in \mathbb{R}^{6\times1}$ the corresponding virtual strain vector. If the virtual displacement field is chosen as the one that is induced by the first load system $\boldsymbol{f}^1$ and the forces and the stresses are due to the second load system $\boldsymbol{f}^2$ which obeys the boundary condition of the system, one has that

$$\sum_{j=1}^{l} f_j^2 x_j^1 = \int_{\mho_p} (\boldsymbol{\sigma}^2)^T \boldsymbol{\varepsilon}^1 d\mho_p \tag{2}$$

where x_j^1 is the displacement at DOF j corresponding to the first load system. This equation shows that $\boldsymbol{x}^1$ is only dependent on the stress-strain relationship inside $\mho_p$. Assume that the structure is linear elastic and that $\boldsymbol{\sigma}^1$ is proportional to $\boldsymbol{\varepsilon}^1$ with stiffness constant K. If $\sum_{j=1}^{l} f_j^2 x_j^1$ is calculated before and after damage has occurred, one has

$$\frac{\sum_{j=1}^{l} f_j^2 x_j^1}{\sum_{j=1}^{l} f_j^2 x_{jd}^1} = \frac{\int_{\mho_p} (\boldsymbol{\sigma}^2)^T \dfrac{\boldsymbol{\sigma}^1}{K} d\mho_p}{\int_{\mho_p} (\boldsymbol{\sigma}^2)^T \dfrac{\boldsymbol{\sigma}_d^1}{K + \Delta K} d\mho_p} = \frac{K + \Delta K}{K} \tag{3}$$

where ΔK is the change in the stiffness parameter in $\mho_p$ due to damage. It is assumed that ΔK is constant within $\mho_p$.

Consider a beam structure under the load configuration $\boldsymbol{f}^1$ as shown in Figure 1. Other than the lateral force, the momental force is applied to the beam. If shear deformation can be neglected and EI is constant between equidistant points $j-1$ and $j+2$, the force configuration of Figure 1 causes nonzero stresses between points $j-1$ and $j+2$ only, whatever the beam is isostatic or hyperstatic. This can be proved if

(1) The vector sum of all forces of Figure 1 is zero;
(2) The resulting moment of all forces of Figure 1 at points $j-1$ and $j+2$ is zero.
(3) The relative rotation between points $j-1$ and $j+2$, due to the force configuration, is zero.

Checking the first two conditions is trivial. The third condition can also be easily checked by means of the virtual work principle with applying a virtual unit moment pair at points $j-1$ and $j+2$.

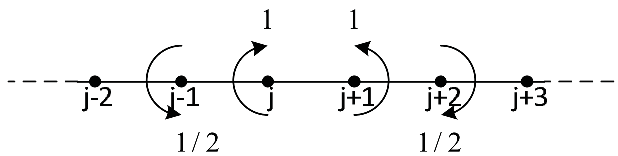

Figure 1: A beam structure with load configuration that causes virtual stresses and strains around one particular element only.

The second load configuration can be chosen as any configuration that obeys the boundary conditions, like for example the configuration of Figure 2.

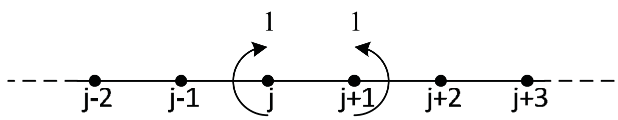

Figure 2: A beam structure with possible second load configuration.

Following Eq. (2), with applying load configuration $\boldsymbol{f}^1$ as shown in Figure 1 and applying load configuration $\boldsymbol{f}^2$ as shown in Figure 2, one has that

$$x_j^1 - x_{j+1}^1 = \int_{\mho_p} M^2 \frac{M^1}{EI} d\mho_p \tag{4}$$

It follows from (3) that

$$\frac{x_j^1 - x_{j+1}^1}{x_{jd}^1 - x_{j+1,d}^1} = \frac{EI + \Delta EI}{EI} \tag{5}$$

It should be noted that for isostatic beams, as an alternative to the load configuration $\boldsymbol{f}^1$ of Figure 1, the force configuration of Figure 2 can be applied. The proof is trivial since for isostatic structures, it is not necessary that the relative rotation between points j and $j+1$ be zero in order to have nonzero stress between theses points only.

The displacement vector $\boldsymbol{x}^1$ under the first load system $\boldsymbol{f}^1$ can be obtained using the following equation

$$\boldsymbol{x}^1 = \boldsymbol{H}\boldsymbol{f}^1 \tag{6}$$

where $\boldsymbol{H}$ is the flexibility matrix. The flexibility matrix can be derived from the relationship between stiffness matrix $\boldsymbol{K}$ and flexibility matrix as

$$\boldsymbol{H} = \boldsymbol{K}^{-1} = \boldsymbol{\Phi}\boldsymbol{\omega}^{-2}\boldsymbol{\Phi}^T = \sum_{i=1}^{N} \boldsymbol{\Phi}_i \omega_i^{-2} \boldsymbol{\Phi}_i^T \tag{7}$$

Where $\mathbf{\Phi}$ is the matrix of mass-normalized mode shapes, $\boldsymbol{\omega}$ is the diagonal matrix of eigenfrequencies and N is the total number of modes. If only the first n modes are available, then the flexibility matrix is truncated. Note that the contribution of the modes in the flexibility is proportional to ω_i^{-2}, the influence of the higher modes is much smaller than the one of lower modes. As a results, the number of truncated modes needed to approximate a non-truncated flexibility matrix is much smaller than the ones needed to approximate a non-truncated stiffness matrix. This benefits the practical cases where only lower modes can be identified with good accuracy.

Because the momental force is utilized in this paper, the mode shapes of corresponding rotary displacement needed to be measured in order to construct the flexibility matrix. This can be achieved by employing the macro-strain mode shapes proposed by Li & Wu (2007). By attaching a long-gauge FBG sensor onto the surface of a beam element between DOF j and DOF $j+1$, the macro-strain measured by an FBG sensor of gauge length l can be expressed as

$$\varepsilon = \frac{h}{l}(\theta_j - \theta_{j+1}) \tag{8}$$

where h is the distance between the inertia axis of the FBG sensor and inertia axis of the beam; θ_j is the rotary displacement at DOF j. Therefore, the difference of the rotary displacement between any two DOFs can be obtained if the macro-strain between these two DOFs is measured. Similarly, the difference of the mode shape of rotary displacement can be obtained if the macro-strain mode shapes are identified from the measured macro-strain signals. The mode shape of rotary displacement can be finally obtained if enough boundary conditions of the rotary displacement or the lateral displacement are known. For instance, the rotary displacement of the fixed end is zero for a cantilever beam, hence the rotary-displacement mode shapes at every DOF can be calculated. Similarly, for a simple support beam, the relative lateral displacement of the two supports is zero if no settlement of these two supports is taken place.

NUMERICAL STUDIES

A numerical simply supported beam was constructed via ANSYS software to verify the proposed idea. The dimension of the beam is 0.03m×0.01m×1.5m, and the number of mesh is 6, 4 and 300 along these dimensions respectively. The element type is 3D elastic solid element with 8 nodes. The elastic modulus, the Poisson ratio and the density of the finite element model are 2.0×10^{11} N/m^2, 0.33 and 7.8×10^3 kg/m^3, respectively. It is assumed that 10 long-gauge FBG sensors were installed on the bottom of the beam to monitor the beam segments labeled as S1 to S10 as shown in Figure 3. Therefore, the longitudinal mode shape displacement of the ends of each sensor on the bottom of the beam is utilized to calculate the macro strain mode shapes.

Four different damage cases were considered in this study as shown in Figure 3. Damage Case 1 is a symmetrical single location damage case where the width of the beam within S5 and S6 sensor range is reduced to 20mm. Damage Case 2 is an unsymmetrical single damage case where the width within S3 range is reduced to 20mm. Damage Case 3 is a multi-damage-locations case mixed by the first 2 damage cases. Damage Case 4 is a continuous damage case where the width within S1 to S8 range is reduced to 20mm.

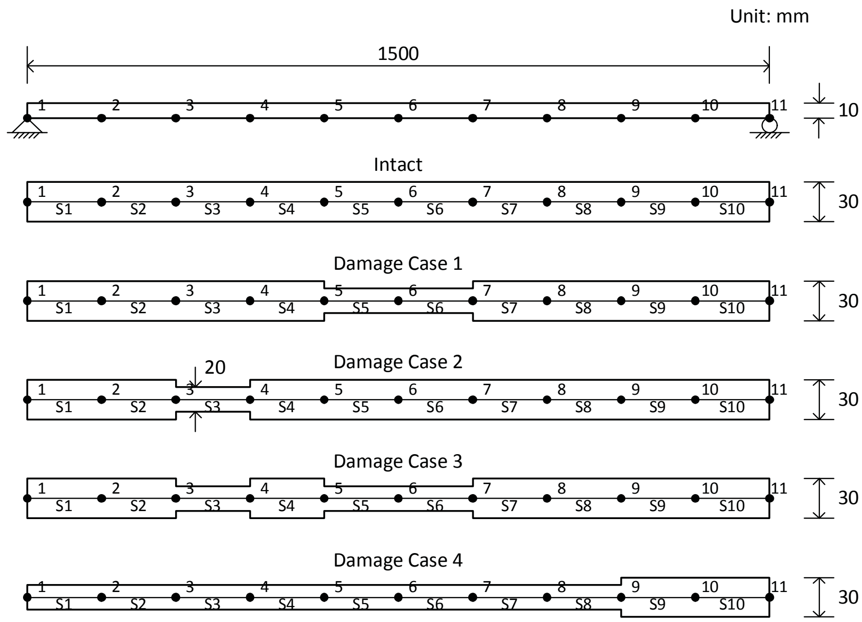

Figure 3 A Simply supported beam model.

The flexibility matrices of different cases were calculated utilizing the rotary mode shapes and the natural frequencies obtained from the numerical model. Practically, the number of qualified fundamental modes identified from measured vibration signals is limited. Therefore in this study, the numbers of the lowest fundamental modes n considered are 1, 2, 3, 5 and 10 in order to see the effects on damage detection results caused by truncation of modes when constructing the flexibility matrices. For each segment, the force configuration of Figure 2 was utilized as both the first load configuration $\boldsymbol{f}^1$ and the second load configuration $\boldsymbol{f}^2$. The flexural rigidity ratios $(EI + \Delta EI) / EI$ of different damage cases utilizing different number of modes as well as the real flexural rigidity ratios are illustrated in Figure 4.

It can be seen from Figure 4 that, in general, the flexural rigidity ratios estimated utilizing the first few modes can not only locate the damage locations but also quantify the damage with acceptable accuracy. It is worth to be noted that even if only the first mode was utilized, the flexural rigidity ratios within the damage zones were estimated quite close to the real value. Furthermore, the methodology seems effective for either symmetrical/unsymmetrical damage or single/multiple/continuous damage cases.

The natural frequencies and mode shapes identified from measured vibration signals may contain errors, and the flexibility matrix is constructed utilizing these modal parameters. Therefore, the estimated flexural rigidity ratio could be altered by these errors in the identified mode shapes. In this study, the noise effect of the modal parameters was investigated. Random noise with noise level 2%, 5% and 10% (in standard deviation) was added directly to the natural frequencies and macro-strain mode shapes both for intact and damaged cases. The first 3 lowest fundamental modes were utilized to construct the flexibility matrix, *i.e.* $n=3$, which is practical for most of the real cases. The flexural rigidity ratio was estimated 1000 times for each noise level and then the mean and standard deviation of the estimated flexural rigidity ratio were calculated. Figures 5(a) to 5(c) illustrate the estimated flexural rigidity ratio of Damage Case 2 considering different noise levels. It can be observed that higher noise level induced higher estimation error both biased and in standard deviation. The error in modal parameters with noise level higher than 10% caused the damage localization and quantification not possible in Damage Case 2 where the real flexural rigidity ratio of S3 equaled to 2/3. Similar phenomena were observed in other damage cases which are not shown in this paper.

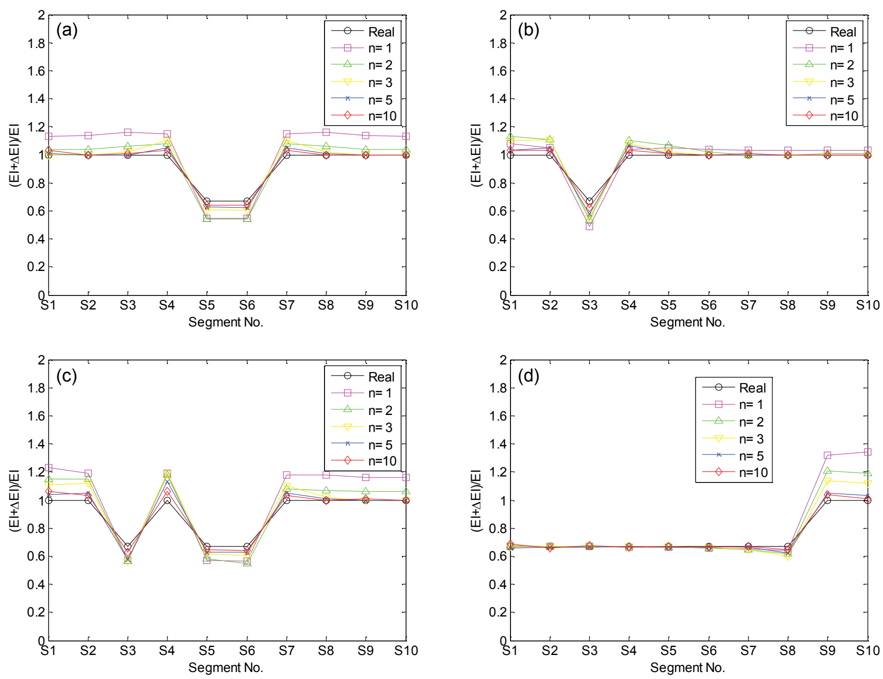

Figure 4 Estimated flexural rigidity ratio utilizing different number of modes for (a) Damage Case 1; (b) Damage Case 2; (c) Damage Case 3; (d) Damage Case 4.

In addition, random noise with noise level 2% was added directly to the natural frequencies and lateral displacement mode shapes both for intact and damaged cases. The lateral displacement mode shapes were assumed measured at the 11 nodes as shown in Figure 3. Utilizing corresponding load configuration (-1/2 at j-1 node, 1 at j node and -1/2 at j+1 node) as both the first and second load configurations, the flexural rigidity ratio was estimated utilizing the first 3 lowest fundamental modes with 1000 times. The mean and standard deviation of the estimated flexural rigidity ratio at the j^{th} node were calculated and shown in Figure 5(d). It is obviously that the damage localization and quantification become not possible even with only 2% noise level in the lateral displacement mode shapes. Similar phenomena were observed in other damage cases which are not shown in this paper. Note that in real application, the noise level of macro-strain mode shapes and lateral displacement mode shapes depends on the measurement conditions.

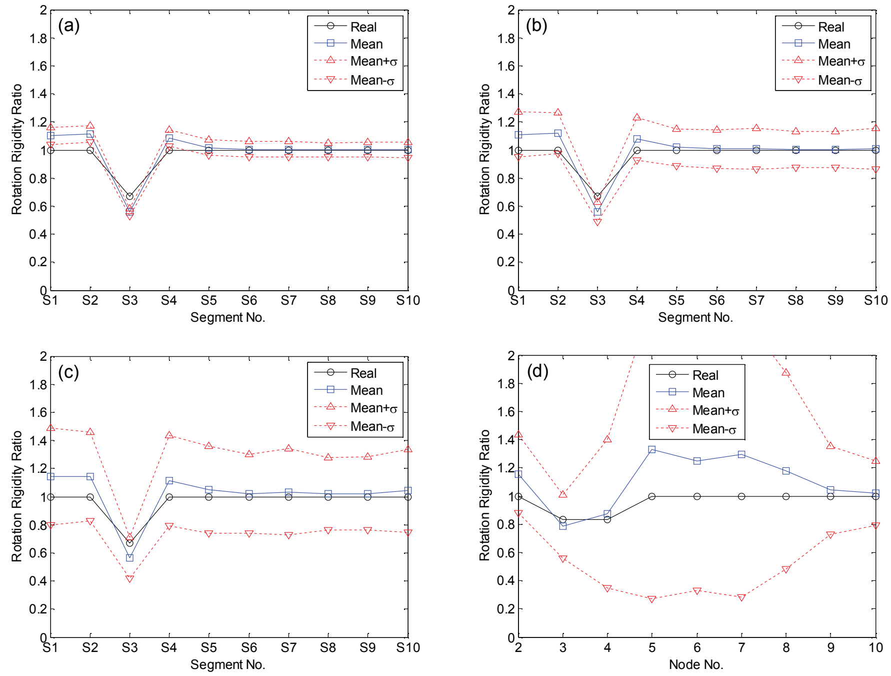

Figure 5 Estimated flexural rigidity ratio of Damage Case 2 utilizing 3 modes for noise level of modal parameters equals to (a) 2%; (b) 5%; (c) 10%; (d) 2% but utilizing lateral displacement mode shapes.

CONCLUSIONS

In this study, the idea to utilize macro-strain measurement via the local flexibility method to perform damage localization and quantification of a beam structure is proposed. Numerical studies considering practical issues including the limited number of structural modes and noise effect in the modal parameters were investigated. The results illustrate potential feasibility of the proposed idea and the potential advantage of utilizing macro-strain mode shapes over the lateral displacement mode shapes in noisy conditions. However, further experimental research is necessary to verify the applicability of the proposed approach to real structures.

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