Principal Component Analysis vs. Independent Component Analysis for Damage Detection

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ABSTRACT

In previous works, the authors showed advantages and drawbacks of the use of PCA and ICA by separately. In this paper, a comparison of results in the application of these methodologies is presented. Both of them exploit the advantage of using a piezoelectric active system in different phases. An initial baseline model for the undamaged structure is built applying each technique to the data collected by several experiments. The current structure (damaged or not) is subjected to the same experiments and the collected data are projected into the models. In order to determine whether damage exists or not in the structure, the projections into the first and second components using PCA and ICA are depicted graphically. A comparison between these plots is performed analyzing differences and similarities, advantages and drawbacks. To validate the approach, the methodology is applied in two sections of an aircraft wing skeleton powered with several PZTs transducers.

INTRODUCTION

Monitoring aircraft structures is a very important task, since knowing continuously the state of the structure provides safety in its normal service. This monitoring is usually performed by Non-Destructive Techniques (NDT), which involves analysis of the signals collected from the sensors attached to the structure being inspected. The paradigm of damage detection (comparison between data from healthy structure and the current structure) can be tackled as a pattern recognition problem.

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Due to the large quantity of experiments and signals that can be gathered from the structure under test, it is necessary to use multivariable techniques for data reduction and pattern recognition. Among others, statistical methodologies based either on Principal Component Analysis (PCA) or Independent Component Analysis (ICA) are very useful. While the goal in PCA is to find an orthogonal linear transformation that maximizes the variance of the variables, the goal of ICA is to find the linear transformation, which the basis vectors are statistically independent and non-Gaussian. Unlike PCA, the basis vectors in ICA are neither orthogonal nor ranked in order. In previous works, authors [1, 2] showed the utility of using ICA and PCA for damage detection. These works use subspace projection techniques (either PCA or ICA) to build a baseline model (linear transformation to a new subspace basis vectors) using data from healthy structure. To detect damages, data from the current structure are projected onto the baseline models. By analyzing these projections, the presence of damages can be known.

In this work, an analytical and experimental comparison between PCA and ICA by embedding both techniques in the same methodology is performed. Two sections of an aircraft wing skeleton, which are powered with six piezoelectric transducers (PZT's) are used to validate the comparison. To include damages in the specimen, three simulated damages were defined by means of adding a mass in different locations. In general terms, this paper includes a theoretical background of PCA and ICA, and the description of the methodology and experimental setup. Furthermore, results are presented and discussed. Finally, conclusions are drawn.

THEORETICAL BACKGROUND

Principal Component Analysis (PCA)

Principal Component Analysis (PCA) is a technique of multivariable and megavariate analysis which may provide arguments for reducing a complex data set to a lower dimension and reveal some hidden and simplified structure/patterns that often underlie it [3]. The main goal of Principal Component Analysis is to obtain the most important characteristics from data. In order to develop a PCA model, it is necessary to arrange the collected data in a matrix **X.** This $m \times n$ matrix contains information from n sensors and m experimental trials [4]. Since physical variables and sensors have different magnitudes and scales, each data-point is scaled using the mean of all measurements of the sensor at the same time and the standard deviation of all measurements of the sensor. Once the variables are normalized, the covariance matrix $\boldsymbol{C_x}$ is calculated. It is a square symmetric $m \times m$ matrix that measures the degree of linear relationship within the data set between all possible pairs of variables (sensors). The subspaces in PCA are defined by the eigenvectors and eigenvalues of the covariance matrix as follows:

$$\mathbf{C}_x \tilde{\mathbf{P}} = \tilde{\mathbf{P}} \Lambda \tag{1}$$

Where the eigenvectors of $\mathbf{C_x}$ are the columns of $\tilde{\mathbf{P}}$, and the eigenvalues are the diagonal terms of $\boldsymbol{\Lambda}$ (the off-diagonal terms are zero). Columns of matrix $\tilde{\mathbf{P}}$ are sorted according to the eigenvalues by descending order and they are called as (by some authors) *Principal Components* of the data set or loading vectors. The eigenvectors with the highest eigenvalue represents the most important pattern in the data with the largest quantity of information. Choosing only a reduced number $r < n$ of principal components, those corresponding to the first eigenvalues, the reduced transformation matrix could be imagined as a model for the structure. In this way, the new matrix **P** ($\tilde{\mathbf{P}}$ sorted and reduced) can be called as PCA model. Geometrically, the transformed data matrix **T** (score matrix) represents the projection of the original data over the direction of the principal components **P**:

$$\mathbf{T} = \mathbf{X}\mathbf{P} \tag{2}$$

In the full dimension case (using $\tilde{\mathbf{P}}$), this projection is invertible (since $\tilde{\mathbf{P}}\tilde{\mathbf{P}}^{\mathrm{T}} = \mathbf{I}$) and the original data can be recovered as $\mathbf{X} = \mathbf{T}\tilde{\mathbf{P}}^{\mathrm{T}}$. In the reduced case (using **P**), with the given **T**, it is not possible to fully recover **X**, but **T** can be projected back onto the original *m*-dimensional space and obtain another data matrix as follows:

$$\hat{\mathbf{X}} = \mathbf{T}\mathbf{P}^{\mathrm{T}} = (\mathbf{X}\mathbf{P})\mathbf{P}^{\mathrm{T}} \tag{3}$$

Therefore, the residual data matrix (the error for not using all the principal components) can be defined as the difference between the original data and the projected back.

$$\begin{aligned} \mathbf{E} &= \mathbf{X} - \hat{\mathbf{X}} \\ &= \mathbf{X} - \mathbf{X}\mathbf{P}\mathbf{P}^{\mathrm{T}} \\ &= \mathbf{X}\left(\mathbf{I} - \mathbf{P}\mathbf{P}^{\mathrm{T}}\right) \end{aligned} \tag{4}$$

Independent Component Analysis (ICA)

ICA is a statistical technique very useful in systems involving multivariable data. The general idea is to change the space from an *m*-dimensional to an *n*-dimensional space such that the new space with the transformed variables (components) describes the essential structure of the data containing the more relevant information from the sensors. Among its virtues is that ICA has a good performance in pattern recognition, noise reduction and data reduction. The goal of ICA is to find new components (new space) that are mutually independent in complete statistical sense. Once the data are projected into this new space, these new variables have no any physical sense and cannot be directly observed, for that, these new variables are known as *latent variables*. If r random variables are observed (x_1, x_2,…, x_r), they can be modeled as linear combinations of n random variables (s_1, s_2,…, s_n) as follows:

$$\boldsymbol{x_i} = \boldsymbol{t}_{i1}\boldsymbol{s}_1 + \boldsymbol{t}_{i2}\boldsymbol{s}_2 + \ldots + \boldsymbol{t}_{in}\boldsymbol{s}_n\,. \tag{5}$$

Each t_{ij} in (5) is an unknown real coefficient. By definition, the set of s_j should be statistically mutually independent and can be designed as the Independent Components (ICs). In matrix terms, equation (5) can be written as

$$\boldsymbol{x} = \mathbf{T}\boldsymbol{s}, \tag{6}$$

where $\boldsymbol{x} = (x_1, x_2,\ldots, x_r)^{\mathrm{T}}$, $\boldsymbol{s} = (s_1, s_2,\ldots, s_n)^{\mathrm{T}}$ and **T** is the $r \times n$ mixing matrix that contains all t_{ij}. If each random variable x_i consists of time-histories with m data points (m-dimensional), the ICA model still holds the same mixing matrix and it can be expressed as:

$$\mathbf{X} = \mathbf{TS}, \tag{7}$$

where $\mathbf{X}$ is the $r \times m$ matrix that contains the observations. Each row of $\mathbf{X}$ represents the time histories. $\mathbf{S}$ is the Independent Component matrix, where each column is the vector of latent variables of each original variable. Since $\mathbf{T}$ and $\mathbf{S}$ are unknown, it is necessary to find these two elements considering that only the $\mathbf{X}$ matrix is known. The ICA algorithm finds the independent components by minimizing or maximizing some measure of independence [5]. To perform ICA, the first step includes the application of pre-whitening to the input data $\mathbf{X}$. The main idea is to use a linear transformation to produce a new data matrix $\mathbf{Z}=\mathbf{VX}$ whose elements are mutually uncorrelated and their variances equal unity. It means that the covariance matrix of $\mathbf{Z}$ is the identity matrix ($E\{\mathbf{ZZ}^{\mathbf{T}}\}=\mathbf{I}$). A popular method to obtain the whitening matrix $\mathbf{V}$ is by means of Singular Value Decomposition (SVD), such as the one used in Principal Component Analysis (PCA) and it is given by:

$$\mathbf{V} = \boldsymbol{\Lambda}^{-1}\mathbf{P}^{\mathbf{T}}, \tag{8}$$

where the eigenvectors of the covariance matrix $\mathbf{ZZ}^{\mathbf{T}}$ are the columns of $\mathbf{P}$ and the eigenvalues are the diagonal terms of Λ (the off-diagonal terms are zero). The second step is to define a separating matrix $\mathbf{W}$ that transforms the matrix $\mathbf{Z}$ to the matrix $\mathbf{S}$ whose variables are non-Gaussian and statistically independent:

$$\mathbf{S} = \mathbf{W}^{\mathbf{T}}\mathbf{Z}. \tag{9}$$

There are several approaches to reach this goal. Maximizing the non-gaussianity of $\mathbf{W}^{\mathbf{T}}\mathbf{Z}$ give us the independent components. On the other hand, minimizing the mutual information between the columns of $\mathbf{W}^{\mathbf{T}}\mathbf{Z}$ is to minimize the dependence between them. The non-gaussianity can be measured by different methods, kurtosis and negentropy being the most commonly used. The first one is sensitive to outliers and the other is based on the information theory quantity of entropy. In this paper, a brief explanation about negentropy is included. If the reader is interested in to know other approaches, an excellent summary is presented by Hyvarinen et al. in [5]. The differential entropy of a continuous-valued random vector s_i with probability density function $p(s_i)$ is interpreted as the degree of information that s_i gives and it is defined in the form:

$$\boldsymbol{H}(\boldsymbol{s}_i) = -\int \boldsymbol{p}(\boldsymbol{s}_i)\log(\boldsymbol{p}(\boldsymbol{s}_i))\boldsymbol{ds}_i. \tag{10}$$

A Gaussian variable has maximum entropy among all random variables with equal variance. In this way, a modified version of the entropy (called negentropy) can be used to measure the non-gaussianity of the variables. Negentropy $J(s_i)$ is defined as:

$$\boldsymbol{J}(\boldsymbol{s}_i) = \boldsymbol{H}(\tilde{\boldsymbol{s}}_i) - \boldsymbol{H}(\boldsymbol{s}_i), \tag{11}$$

where $\tilde{\boldsymbol{s}}_i$ is a Gaussian random variable with the same covariance matrix as s_i. Negentropy is always non-negative, and it is zero only if s_i has Gaussian distribution.

Data projection into the model

Data from the healthy structure are used to build the either PCA or ICA baseline model. The transformation matrix for PCA, the $\mathbf{P}$ matrix (principal components or loading vectors) is calculated by means of Singular Value Decomposition algorithm.

On the other hand, the transformation matrix for ICA, the **S** matrix (independent components) is calculated by using the FastICA Matlab package developed by the University of Helsinki [7]. New data from the unknown structure state $\mathbf{X_c}$ is projected into the models. In the case of PCA, the score matrix **T** can be calculated by means of the equation (2). In the case of ICA, the mixing matrix **T** can be calculated from equation (7). Since $\mathbf{SS^T}$=**I** (because of the properties of **S)**, this can be rewrote as follows:

$$\mathbf{X}_c\mathbf{S}^T = \mathbf{T}\mathbf{S}\mathbf{S}^T \qquad (12)$$

$$\mathbf{X}_c\mathbf{S}^T = \mathbf{T} \quad ,$$

DAMAGE DETECTION METHODOLOGY

The damage detection methodology by using (either PCA or ICA) was previously proposed by the authors [2,6]. This includes the use of an active piezoelectric system in different phases, where each phase is defined by the excitation of a piezoelectric and obtaining the signals from the other sensors attached to the structure. Using data from healthy structure, the calculation of a (either PCA or ICA) baseline model for each phase is performed using the equations presented in the theoretical background. After building the baseline model for each actuator, signals from the current structure are projected into the model and two of these projections are plotted and analyzed to determine if exist a damage on the structure (see Figure 1).

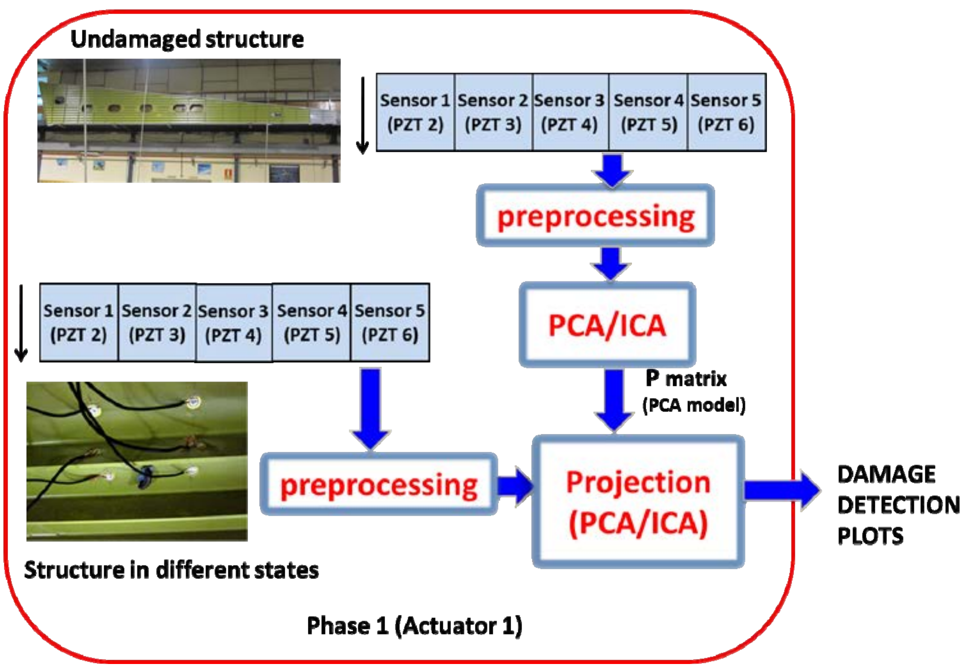
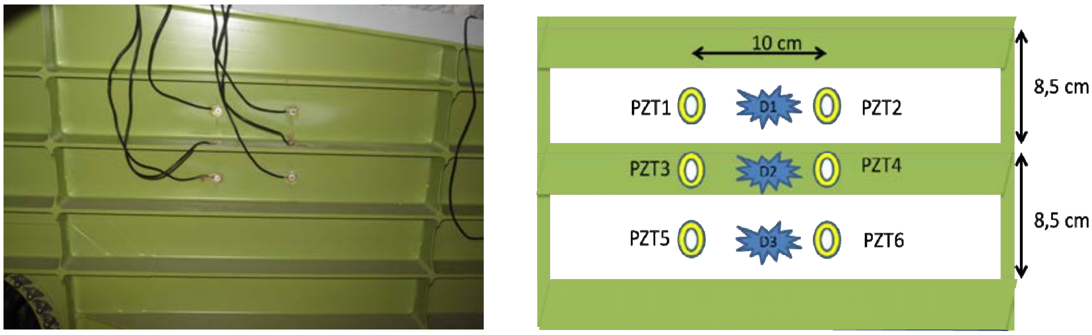


Figure 1. Damage detection methodology using (either PCA or ICA) and the PZT 1 as actuator.

EXPERIMENTAL SETUP

This work involves experiments with an aircraft wing skeleton. In general, this structure is divided in small sections by using stringers and ribs. For testing the approaches, two sections of the wing structure were used (Figure 2a). These sections are powered by 6 PZT's, two in upper section, two in lower section and two in the stringer. Four different states including the healthy structure are analyzed. By adding a mass in different locations, three damages were simulated as shown in Figure 2b.



a. b.

Figure 2. PZT's location and damage description.

RESULTS

From Figure 3, it can be shown results obtained in phases 1, 3, 4 and 5. At the right side results using PCA are graphically depicted, at the left side, results using ICA. Each shape represents the state of the structure, in this way; undamaged structure is represented by the green plus sign, damage 1 by the magenta circles, damage 2 by the red diamonds, and damage 3 by the cyan asterisk.

As can be observed in the plots, in both methodologies, damages are clearly distinguished from undamaged structure; additionally, some phases are more sensitive than others in both methods. Other important characteristic is related with the possibility to distinguish between damages. Although, there is one damage in the stringer which is detected by the sensors attached in the same stringer, this is also detected by using both methodologies for other phases as in the phase 1, this means that the combined analysis of the sensors (using all phases) is a very useful tool because allows to consider the dynamic responses in the whole sensor network. Although there are differences in the scale of the graphs of ICA and PCA, it is possible to see that into the PCA plots there is a clearer separation between each data set when only two projections are used. This is because with PCA it is possible to ensure that these components contain the most relevant information with maximal variance, while with ICA is not possible to define which components are more relevant directly from the algorithm.

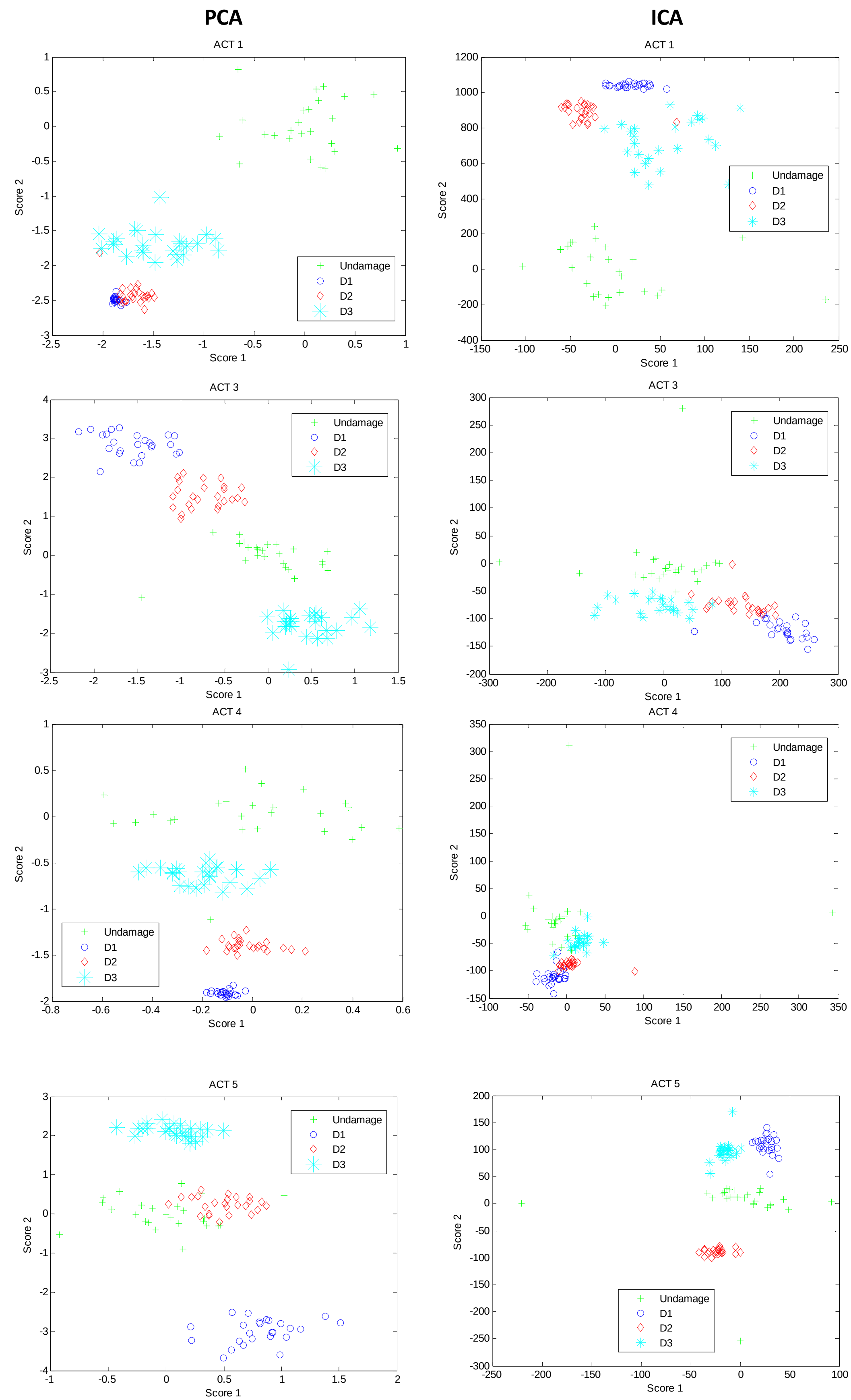


Figure 3. Damage detection plots using phases 1, 3, 4 and 5.

CONCLUSIONS

In this work a comparison between the results obtained for two methodologies in damage detection (PCA and ICA) using data driven from a Piezoelectric active system, were shown. Both methodologies allowed detecting the damages showing in most cases a clear distinction between the data from undamaged structure and the other three damage states. The results can change depending of the phase to being analyzed but in all cases it is possible to distinguish the presence of damage. In addition, it was shown some differences between the results using both methodologies, for instance the definition of the number of Components or the possibility of define different data set by identifying the kind of damage due to the separation that is possible to see in some of the phases.

An important difference between PCA and ICA is related to the number of components used in each methodology, in the PCA case this number can be determined by the variance criteria, but in the ICA case don't exist a criteria for determining how many components represent the dynamic of the data, despite this, was showed that with just two components is possible to define the presence of damages, of course is necessary to evaluate all the combinations to determine which components show better results. One way to improve the results with ICA could be using another tool that includes all the Independent Components by each phase or performing data fusion including the components from all phases.

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